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A THEOREM ON APPROXIMATION BY ALGEBRAIC POLYNOMIALS OF FUNCTIONS WITH GIVEN GENERALISED MODULUS OF SMOOTHNESS

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There are well known theorems on approximation by algebraic polynomials of functions, where the generalised modulus of smoothness is defined by means of symmetric operator of generalised translation. It is of interest obtaining the same results for a modulus of smoothness defined by an asymmetric operator of generalised translation.

We introduce such an operator, define the generalised modulus of smoothness by its means, and obtain the direct and inverse theorems in approximation theory for it.

Denote by $L_{p,\alpha}$, $1 \le p \le \infty$, the set of functions f such that $f(x)\left(1-x^2\right)^{\alpha} \in L_p$, and put

$$\|f\|_{p,\alpha} = \left\| f(x)\left(1-x^2\right)^{\alpha} \right\|_p .$$

By $E_n(f)_{p,\alpha}$ we denote the best approximation of a function $f \in L_{p,\alpha}$ by algebraic polynomials of degree not greater than $n-1$, in $L_{p,\alpha}$ metrics.

For a function $f \in L_{p,\alpha}$ we define the asymmetric operator of generalised translation:

$$\begin{aligned}\hat{\tau}_t(f,x) = {} & \frac{1}{\pi\left(1-x^2\right)\cos^4 t/2} \\ & \times \int_0^{\pi} \left\{ 2\left(\sqrt{1-x^2}\cos t + x\sin t\cos\varphi + \sqrt{1-x^2}(1-\cos t)\sin^2\varphi\right)^2 \right. \\ & \left. - 1 + \left(x\cos t - \sqrt{1-x^2}\sin t\cos\varphi\right)^2 \right\} f\left(x\cos t - \sqrt{1-x^2}\sin t\cos\varphi\right) d\varphi.\end{aligned}$$

By means of that operator of generalised translation we define the generalised modulus of smoothness by

$$\hat{\omega}\left(f,\delta\right)_{p,\alpha} = \sup_{|t|\le\delta} \left\|\hat{\tau}_t\left(f,x\right) - f(x)\right\|_{p,\alpha} .$$

Theorem 1. *Let given numbers p and α be such that $1 \le p \le \infty$;*

$$\begin{aligned} 1/2 < \alpha \le 1 \qquad & \text{for } p = 1, \\ 1 - \frac{1}{2p} < \alpha < \frac{3}{2} - \frac{1}{2p} \qquad & \text{for } 1 < p < \infty, \\ 1 \le \alpha < 3/2 \qquad & \text{for } p = \infty. \end{aligned}$$

Let $f \in L_{p,\alpha}$. For every natural number n the following inequalities hold

$$C_1 E_n(f)_{p,\alpha} \le \hat{\omega}\left(f, 1/n\right)_{p,\alpha} \le C_2 \frac{1}{n^2} \sum_{\nu=1}^{n} \nu E_\nu(f)_{p,\alpha},$$

where the positive constants C_1 and C_2 do not depend on f and n.

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