Techniques of Differentiation

F. M. Berisha

UNIVERSITETI I EJL
ЈИЕ УНИВЕРЗИТЕТ
SEE UNIVERSITY

South East European University, Tetovo

Aims and Objectives

- Developing differentiation techniques
- Using the developed techniques to differentiate a polynomial
- Applying the techniques in a business application

Contents

Differentiating a Constant Function

The Constant Rule

For any constant c,

$$\frac{d}{dx}(c) = 0.$$

That is, the derivative of a constant is zero.

The Graph of the Function $y = c$

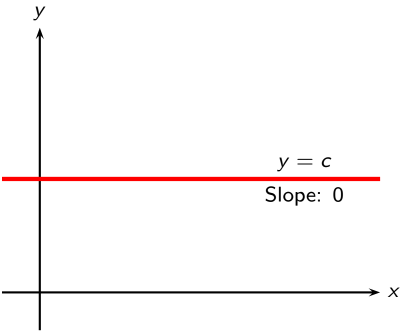

Figure: The graph of $f(x) = c$.

Example

Example

Find the derivative of the function $f(x) = -5$.

Solution.

We have

$$\frac{d}{dx}(-5) = 0.$$

$\square$

Differentiating a Power Function

The Power Rule

For any real number n,

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

In words, to find the derivative of x^n,
reduce the exponent n of x by 1
and multiply be the original exponent.

Examples of Using the Power Rule

Example

Find

1. $\frac{d}{dx}(x^5)$
2. $\frac{d}{dx}(\sqrt{x})$
3. $\frac{d}{dx}\left(\frac{1}{x^2}\right)$

Solution. . .

1. $$\frac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$$

$\square$

Examples of Using the Power Rule. (Continued)

. . . Solution.

2. $$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

3. $$\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}(x^{-2}) = -2x^{-2-1} = -2x^{-3} = -\frac{2}{x^3}$$

□

Differentiating a Constant Multiple

The Constant Multiple Rule

If c is a constant and $f(x)$ is differentiable, then so is $cf(x)$ and

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}[f(x)].$$

That is, the derivative of a multiple is the multiple of the derivative.

Examples of Using the Multiple Rule

Example

Find

1. $\frac{d}{dx}(5x^4)$
2. $\frac{d}{dx}(-\frac{3}{\sqrt{x}})$

Solution...

1. $$\frac{d}{dx}(5x^4) = 5\frac{d}{dx}(x^4) = 5(4x^3) = 20x^3$$

□

Examples of Using the Multiple Rule. (Continued)

. . . Solution.

2.
$$\frac{d}{dx}(-\frac{3}{\sqrt{x}}) = \frac{d}{dx}(-3x^{-1/2}) = -3\left(-\frac{1}{2}x^{-3/2}\right) = \frac{3}{2}x^{-3/2}$$

$\square$

Differentiating a Sum and a Difference of Functions

The Sum and Difference Rules

If $f(x)$ and $g(x)$ are differentiable,
then so are their sum and difference, and

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)],$$
$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)];$$

i.e.,

$$(f+g)' = f' + g' \quad \text{and} \quad (f-g)' = f' - g'$$

In words, the derivative of a sum or a difference
is the sum or the difference of the derivatives.

Examples of Using the Sum and Difference Rules

Example

Find

1. $\frac{d}{dx}(x^{-3}+5)$
2. $\frac{d}{dx}(2x^7-3x^{-5})$

Solution...

1. $$\frac{d}{dx}(x^{-3}+5)=\frac{d}{dx}(x^{-3})+\frac{d}{dx}(5)=-3x^{-4}+0=-3x^{-4},$$

$\square$

Examples of Using the Sum and Difference Rules. (Continued)

. . . Solution.

2.
$$\frac{d}{dx}(2x^7 - 3x^{-5}) = 2\frac{d}{dx}(x^7) - 3\frac{d}{dx}(x^{-5})$$
$$= 2(7x^6) - 3(-5x^{-6}) = 14x^6 + 15x^{-6}.$$

□

Differentiating a Polynomial

Example

Find the derivative of the polynomial

$$y = 4x^3 - 5x^2 + 8x - 7.$$

Solution.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(4x^3 - 5x^2 + 8x - 7) \\ &= 4\frac{d}{dx}(x^3) - 5\frac{d}{dx}(x^2) + 8\frac{d}{dx}(x) + \frac{d}{dx}(-7) \\ &= 12x^2 - 10x^1 + 8x^0 + 0 = 12x^2 - 10x + 8.\end{aligned}$$

□

Percentage Rate of Change of a Function

- $$\begin{bmatrix}\text{Percentage rate} \\ \text{of change of } Q\end{bmatrix} = 100\frac{[\text{Rate of change of } Q]}{[\text{Size of } Q]} = 100\frac{Q'(x)}{Q(x)}.$$
- Percentage rate of a yearly change of 200 € in salary
  - for a person earning 100,000 € a year:
    $$100 \cdot \frac{200}{100{,}000} = 0.2$$
  - for a person earning 1,000 € a year:
    $$100 \cdot \frac{200}{1{,}000} = 20$$

An Economics Application

Example

The gross domestic product (GDP) of a certain country was

$$N(t) = t^2 + 2t + 50$$

hundred million euros t years after 2000.

1. At what rate was the GDP changing with respect to time in 2005?
2. At what percentage rate was the GDP changing with respect to time in 2005?

An Economics Application. (Continued)

Solution...

1. The rate of change of the GDP is the derivative

$$N'(t) = 2t + 2.$$

The rate of change in 2005 is

$$N'(5) = 2 \cdot 5 + 2 = 12$$

hundred million euros per year.

□

An Economics Application. (Continued)

. . . Solution.

2. The percentage rate of change of the GDP was

$$100\frac{N'(5)}{N(5)} = 100\frac{12}{5^2+2\cdot 5+50} = 100\cdot\frac{12}{85} \approx 14.1;$$

i.e., approximately 14.1% per year.

□

For Further Reading

- `http://fberisha.netfirms.com`
- Homework: Exercises from teaching materials
- L. D. Hofmann, G. L. Bradley, *Calculus – for business, economics and life sciences,* pp. 109–122.
- F. M. Berisha, M. Q. Berisha, *Matematikë – për biznes dhe ekonomiks,* pp. 161–167.

Summary

- Techniques of differentiation:
  - The constant rule
  - The power rule
  - The constant multiple rule
  - The sum and difference rules
- Differentiating a polynomial
- Percentage rate of change of a function