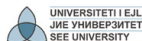


# Ordinary Annuities

F. M. Berisha



South East European University, Tetovo

# Aims and Objectives

- Understanding relationship between the present value of compound interest and ordinary annuities
- Calculating the present value of an ordinary annuity with periodic payments
- Applying in practical applications

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  - Examples of Applications
  
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# Present Value of Ordinary Annuities

- Suppose that a lump sum of money is invested in an *ordinary annuity* that earns interest at rate  $p\%$ , compounded annually, in order to receive (withdraw) payments of size  $R$  at the end of each year.
- Given:
  - $P$  – present value of the ordinary annuity,
  - $R$  – value of a payment (withdrawal),
  - $p$  – annual compound interest rate,
  - $n$  – number of years of duration of the ordinary annuity.

## Present Value of Ordinary Annuities. (Continued)

- The present value of the first payment (payable after 1 period):

$$R \cdot \frac{1}{r}, \quad \text{where is } r = 1 + \frac{p}{100}$$

- The present value of the second payment (payable after 2 periods):

$$R \cdot \frac{1}{r^2}$$

- ...

- The present value of the last payment (payable after  $n$  periods):

$$R \cdot \frac{1}{r^n}$$

## Present Value of Ordinary Annuities. (Continued)

- Hence, the present value of the ordinary annuity is

$$\begin{aligned} P &= R \cdot \frac{1}{r} + R \cdot \frac{1}{r^2} + \cdots + R \cdot \frac{1}{r^n} \\ &= R \cdot \frac{1}{r^n} (r^{n-1} + r^{n-2} + r^{n-3} + \cdots + 1). \end{aligned}$$

- Recall the sum of a geometric sequence:

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{r^n - 1}{r - 1}$$

# Present Value of an Ordinary Annuity with Annual Payments

## Present Value of an Ordinary Annuity

If a payment of  $R$  € is to be made at the end of each year for  $n$  years from an account that earns interest at a rate  $p\%$ , compounded annually, then the account is an ordinary annuity, and its *present value* is

$$P = R \frac{r^n - 1}{r^n(r - 1)},$$

## Present Value with Annual Payments

### Example

What is the lump sum that one must invest in an annuity in order to receive 5,000 € at the end of each year for the next 12 years, if the annuity pays 10%, compounded annually?



# Present Value with Annual Payments. (Continued)

## Solution.

We have  $R = 5,000$ ,  $p = 10$ ,  $n = 12$ .

$$P = R \frac{r^n - 1}{r^n(r - 1)}$$

$$r = 1 + \frac{10}{100} = 1.1$$

$$P = 5,000 \cdot \frac{1.1^{12} - 1}{1.1^{12} \cdot (1.1 - 1)}$$

$$\approx 5,000 \cdot 6.813692 \approx 34,068.46.$$



## Present Value with Periodic Payments

### Example

What is the present value of an annuity of 4,000 € payable at the end of each 6-month period for 12 years if money is worth 8%, compounded semiannually?

# Present Value of an Ordinary Annuity with Periodic Payments

## Present Value of an Ordinary Annuity with Periodic Payments

If a payment of  $R$  € is to be made at the end of each period  $m$  times per year for  $n$  years from an account that earns interest at a rate  $p\%$ , compounded  $m$  times per year, then the *present value* of the ordinary annuity is

$$P = R \frac{r^{mn} - 1}{r^{mn}(r - 1)},$$

where now is

$$r = 1 + \frac{p}{100m}.$$

## Present Value with Periodic Payments. (Continued)

Solution.

Given  $R = 4,000$ ,  $p = 8$ ,  $n = 12$ ,  $m = 2$ ,

$$r = 1 + \frac{8}{100 \cdot 2} = 1.04$$

$$P = 4,000 \cdot \frac{1.04^{2 \cdot 12} - 1}{1.04^{2 \cdot 12} \cdot (1.04 - 1)}$$

$$\approx 4,000 \cdot 15.246963 \approx 60,987.85.$$



## Duration Time of an Ordinary Annuity

### Example

An amount of 87,700 € is invested in a bank at 6%, compounded semiannually. Calculate how many payments of 5,000 € can be received at the end of each half-year.

## Duration Time of an Ordinary Annuity. (Continued)

### Solution.

We have  $P = 87,700$ ,  $R = 5,000$ ,  $p = 6$ ,  $m = 2$ ,

$$r = 1 + \frac{p}{100m} = 1 + \frac{6}{100 \cdot 2} = 1.03.$$

$$87,700 = 5,000 \frac{1.03^{2n} - 1}{1.03^{2n}(1.03 - 1)},$$

$$\frac{877}{50} \cdot 0.03 = \frac{1.03^{2n} - 1}{1.03^{2n}},$$

$$0.5262 \cdot 1.03^{2n} = 1.03^{2n} - 1.$$



## Duration Time of an Ordinary Annuity. (Continued)

Solution.

$$1.03^{2n}(1 - 0.5262) = 1$$

$$1.03^{2n} = \frac{1}{0.4738}$$

$$2n \log 1.03 \approx \log 2.110595$$

$$2n \approx \frac{\log 2.110595}{\log 1.03} \approx 25.27$$

Since  $25 < 2n < 26$ , the annuity will generate 25 complete payments of 5,000 €.



## For Further Reading

- <http://fberisha.netfirms.com>
- **Homework:** Exercises from teaching materials
- D. P. Maki, M. Thompson, *Finite mathematics*, pp. 421–432.
- S. T. Karris, *Mathematics for business, science and technology*, pp. 7-1–7-84.
- F. M. Berisha, M. Q. Berisha, *Matematikë – për biznes dhe ekonomiks*, pp. 85–90.



# Summary

- Relationship between compound interest and calculating the present value of an ordinary annuity.
- Applications of ordinary annuities
  - with a single payment period per year
  - with multiple payment periods per year