Functional Models

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Aims and Objectives

- Representing a practical situation by means of a functional model.
- Applying mathematical models to solving practical application problems.

Contents

Example of Revenue Function

- A *mathematical model* is a mathematical representation of a practical situation

Example

A retailer assumes that the price p, in cents, per unit of a certain commodity when x units are sold satisfies the relation

$$\frac{5}{6}p - 35x = 15.$$

Express the generated revenue as a function of x.

Example of Revenue Function. (Continued)

Solution...

The revenue R generated by selling x units of the commodity for a price p is

$$R = px.$$

To express R as a function of x alone, express p as a function of x:

$$\begin{aligned} \frac{5}{6}p - 35x &= 15 \\ \frac{5}{6}p &= 35x + 15 \\ p &= \frac{6}{5}(35x + 15) \\ p &= 42x + 18 \end{aligned}$$

□

Example of Revenue Function. (Continued)

. . . Solution.

Substitute the obtained expression for p in the formula for R:

$$R(x) = (42x + 18)x = 42x^2 + 18x.$$

$\square$

Proportionality

Proportionality

A quantity Q is said to be:

- *directly proportional* to x if

$$Q = kx$$

  for some constant k;

- *inversely proportional* to x if

$$Q = \frac{k}{x}$$

  for some constant k.

Example of Total Cost Function

Example

At a certain factory, fixed production cost is directly proportional to the number of machines used and variable cost is inversely proportional
to the number of machines used.
Express the total cost as a function of the number of machines used.

Example of Total Cost Function. (Continued)

Solution.

Denote by x the number of machines used
and by $C(x)$ the total production cost.

$$[\text{Fixed Cost}] = k_1 x, \qquad [\text{Variable cost}] = \frac{k_2}{x},$$

where k_1 and k_2 are constants.
Hence,

$$C(x) = k_1 x + \frac{k_2}{x}.$$

The graph of such a function is sketched in the following figure. □

The Graph of a Total Cost Function

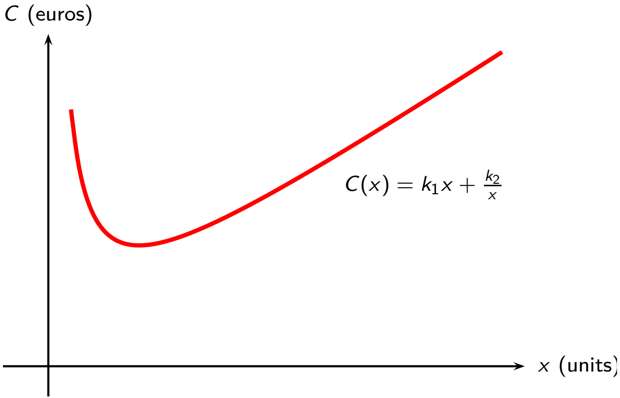

Figure: The total cost as a function of the number of machines used.

Example of Profit Function

Example

A manufacturer can produce blank video tape at a cost of 2 € per cassette. The cassettes have been selling for 3 € apiece, and at that price, consumers have been buying $4,000$ cassettes a month. The manufacturer is planning to raise the price of cassettes and estimates that for each 1 € increase in the price 400 fewer cassettes will be sold each month.

1. Express the manufacturer's monthly profit as a function of the price at which the cassettes are sold.
2. Sketch the graph of the profit function. What price corresponds to maximum profit?

Example of Profit Function. (Continued)

Solution. . .

1. State the desired relationship in words:

$$[\text{Profit}] = [\text{Revenue}] - [\text{Total cost}].$$

Denote by p the price at which each cassette will be sold and by $P(p)$ the corresponding monthly profit.

$\square$

Example of Profit Function. (Continued)

... Solution...

Express the number x of cassettes sold in terms of p.
Since the rate of change is constant
($m = -400$ cassettes for 1 € increase),
the function relating x to p is linear.
Since for $p = 3$ we have $x = 4{,}000$,
the line passes through $(3, 4{,}000)$.
The point-slope form of the equation of the line:

$$\begin{aligned} x - x_0 &= m(p - p_0) \\ x - 4{,}000 &= -400(p - 3) \\ [\text{Number of cassettes sold}] = x &= 5{,}200 - 400p \end{aligned}$$

□

Example of Profit Function. (Continued)

. . . Solution. . .

$$\begin{aligned}[\text{Revenue}] &= R(p) \\ &= p \cdot [\text{Number of cassettes sold}] = p(5,200 - 400p).\end{aligned}$$

$$\begin{aligned}[\text{Total cost}] &= C(p) \\ &= 2 \cdot [\text{Number of cassettes sold}] = 2(5,200 - 400p).\end{aligned}$$

Hence, the total profit is

$$\begin{aligned}P(p) = R(p) - C(p) &= p(5,200 - 400p) - 2(5,200 - 400p) \\ &= (5,200 - 400p)(p - 2) = -400(p - 13)(p - 2) \\ &= -400p^2 + 6,000p - 10,400.\end{aligned}$$

$\square$

The Graph of the Profit Function

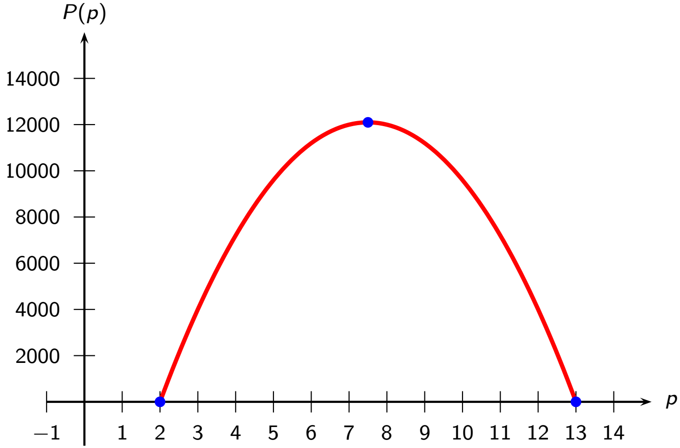

Figure: The profit function $P(p) = -400(p-13)(p-2)$.

Optimization of Profit

. . . Solution.

2. Maximum profit will occur at the value of p corresponding to the vertex of the parabola:

$$p = \frac{-b}{2a} = \frac{-6,000}{2 \cdot (-400)} = 7.5$$

euros.

□

Law of Supply and Demand

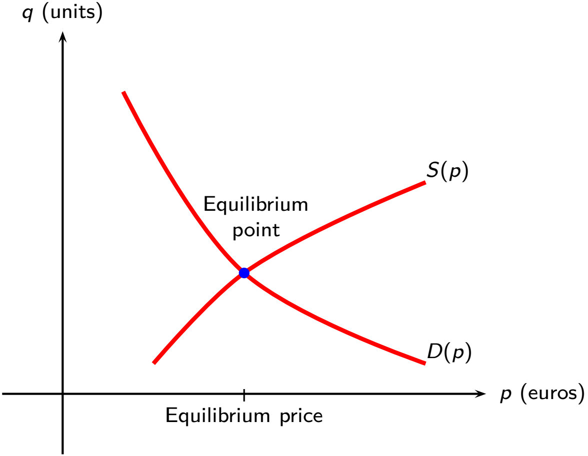

Figure: Market equilibrium: the intersection of supply and demand.

Example of Market Equilibrium

Example

Find the equilibrium price if the supply function for a certain commodity is $S(p) = p^2 + 3p - 70$ and the demand function is $D(p) = 410 - p$.

Example of Market Equilibrium. (Continued)

Solution.

Set $S(p)$ equal to $D(p)$ and solve the equation for p:

$$\begin{aligned} S(p) &= D(p) \\ p^2 + 3p - 70 &= 410 - p \\ p^2 + 4p - 480 &= 0 \\ p_{1/2} &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-480)}}{2 \cdot 1}, \end{aligned}$$

$$p_1 = 20, \quad p_2 = -24.$$

In the application, only the positive values of p are meaningful; hence, we conclude that the equilibrium price is 20 €. □

For Further Reading

- http://fberisha.netfirms.com
- Homework: Exercises from teaching materials
- L. D. Hofmann, G. L. Bradley, *Calculus – for business, economics and life sciences,* pp. 46–61
- F. M. Berisha, M. Q. Berisha, *Matematikë – për biznes dhe ekonomiks,* pp. 133–143

Summary

- Mathematical models
- Models of business functions
  - Revenue function
  - Total cost function
  - Profit function
  - Functions of supply and demand
- Applying functional models for solving practical problems:
  - Optimization
  - Low of supply and demand