Introduction to linear algebra

Systems of two equations

F. M. Berisha

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Aims and Objectives

- Representing graphicly the straight line corresponding to a linear equation with two variables
- Identifying in the equation, understanding and computing the slope of a straight line
- Identifying in the equation the y-intercept of a straight line
- Understanding and calculating the solution of a system of two linear equations

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Contents

1. Linear Equations. Graphical Representation
2. Systems of two linear equations
   - Graphical Solution
   - Analytical Solution

Linear Equations

Shembull

A manufacturer plans to start his own business, manufacturing and selling bicycles.
He wants to compute the *break-even* point;
(the point where the revenues are equal to costs).
He estimates that his *fixed costs* would be $1,000$ € per month, while the *variable costs* will increase *linearly*
(in a straight line fashion).
Preliminary figures show that the variable costs for the production of 500 bicycles will be $9,000$ € per month.

- Total costs for the production of 500 bicycles:

$$\begin{aligned}\text{Total costs} &= \text{Fixed costs} + \text{Variable costs} \\ &= 1000 + 9000 = 10,000.\end{aligned}$$

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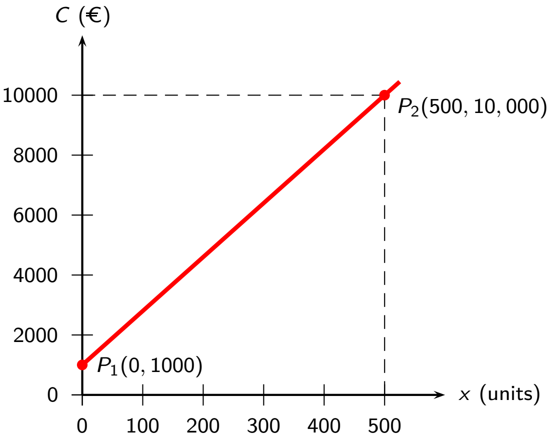

Figura: Total cost versus bicycles sold.

Equation of a Straight Line

- *Cartesian coordinate system*
  - *Abscissa:* the x axis
  - *Ordinate:* the C axis
- The straight line from P_1 to P_2: total costs depending upon the number of bicycles produced
  - $P_1(0, 1000)$
  - $P_2(500, 10000)$
- Equation of a straight line in general:

$$\boxed{y = mx + b}$$

  - x is the abscissa, y is the ordinate
  - m is the *slope*
  - b is the *y-intercept* of the straight line

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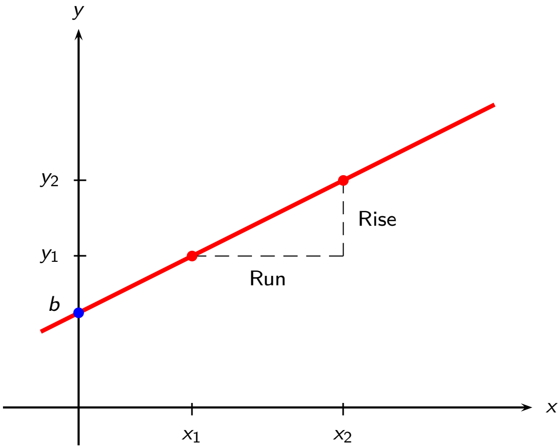

Figura: The straight line $y = mx + b$.

Equation of a Straight Line. (Continued)

- The slope:

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

- In our example:

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{C_2 - C_1}{x_2 - x_1} = \frac{10000 - 1000}{500 - 0} = \frac{9000}{500} = 18$$

- The equation of the total cost line:

$$C = 18x + 1000$$
$$C - 18x = 1000$$

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Graphical Solution of a System of Two Linear Equations

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The manufacturer has determined that the revenue will as well increase linearly, and that if he sells 500 bicycles at 25 € each, he will generate a revenue of $500 \cdot 25 = 12,500$ €. He wants to compute the *break-even* point (where the revenues are equal to costs).

- The new line:
  - starts at $P_3(0,0)$,
  - passes through $P_4(500, 12500)$.

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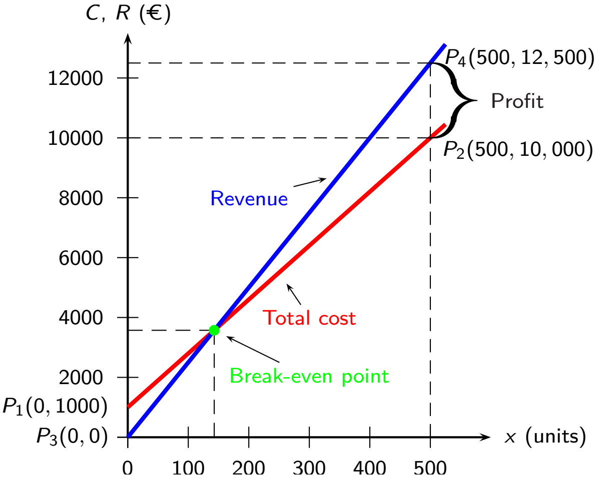

Figura: Intersection of graphs (straight lines) of total cost and revenue.

Analytical Solution of a System of Two Equations

- Equation of the revenue line:

$$R = mx + b$$

- The slope:

$$m = \frac{\text{ngritja}}{\text{ecja}} = \frac{R_2 - R_1}{x_2 - x_1} = \frac{12,500 - 0}{500 - 0} = \frac{125}{5} = 25$$

- The y-intercept:

$$b = 0$$

- The ekuacioni of the revenue:

$$R = 25x$$

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Analytical Solution of a System. (Continued)

- In order to solve anaytically a system, we denote the abscissas and the ordinates of both lines by the same symbols.
  - By x – the number of produced and sold bycicles
  - By y the value of total cost equaling the value of revenue.
- *The system of equations:*

$$y - 18x = 1000$$
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Analytical Solution of a System. (Continued)

- Solve one of the equations for one of the variables and substitute in the other equation:

$$\begin{aligned} 25x - 18x &= 1000 \\ 7x &= 1000 \\ x &= \frac{1000}{7} \end{aligned}$$

- We substitute the obtained value in order to find the other variable:

$$y = 25 \cdot \frac{1000}{7} = \frac{25000}{7}.$$

- In our application:

$$x = \frac{1000}{7} \approx 142.86 \approx 143$$

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