Chapter 1

Numbers and Arithmetic Operations

This chapter is a review of the basic arithmetic concepts. It is intended for readers feeling that they need a math review from the very beginning. It forms the basis for understanding and working with relations (formulas) encountered in business, science and technology. Readers with a fair mathematical background may skip this chapter. Others may find it useful as well as a convenient source for review.

1.1 Number Systems

The *decimal* (*base 10*) number system uses the *digits* (numbers) *0, 1, 2, 3, 4, 5, 6, 7, 8*, and 9. This is the number system we use in our everyday arithmetic calculations such as the monetary transactions. Another number system is the *binary* (*base 2*) that uses the digits *0* and *1* only. The binary system is used in computers and it is being taught in electronics courses. We will not be concerned with the binary system in this text.

1.2 Positive and Negative Numbers

A *positive number* is a number greater than zero and it is understood to have a plus (+) sign in front of it. The (+) sign in front of a positive number is generally omitted. Thus, any number without a sign in front of it is understood to be a positive number. A *negative number* is less than zero and it is written with a minus (–) sign* in front of it. The minus (–) sign in front of a negative number is a must; otherwise it would not be possible to distinguish the negative from the positive numbers. Positive and negative numbers can be whole (integer) or fractional numbers. Several examples will be presented in this chapter to illustrate their designation, how they are added, subtracted, multiplied, and divided with other numbers. To avoid confusion between the addition operation (+) and positive numbers, which are also denoted with the (+) sign, we will enclose positive numbers with their sign inside parentheses whenever necessary. Likewise, we will enclose negative numbers in parentheses to distinguish them from the subtraction (–) symbol. This will be illustrated with the examples that follow.

Example 1.1

Joe Smith's checking account shows a balance of $534.29. Thus, we can say that his balance is *+534.39* dollars but we normally omit the plus (+) sign, and we say that his balance is *534.39* dollars.

* *The financial community, such as banks, usually enclose a negative number in parentheses without the minus sign. Most often, this designation appears in financial statements.*

Example 1.2

Bill Jones, unaware that his checking account has a balance of only *$78.31*, makes a purchase of *$128.74*. He pays this amount with a check. His new account balance is now *–128.74* dollars. Here, the minus (–) sign is a must.

The *absolute value* of a number is that number without a positive or negative sign, and is enclosed in small vertical lines. For example, the absolute value of X is written as $|X|$. The number *0 (zero)* is considered neither positive nor negative; it is the number that separates the negative from the positive numbers. The positive and negative numbers that we are familiar with, are referred to as the *real numbers*[*] and are shown below on the so-called *real axis of numbers*[**].

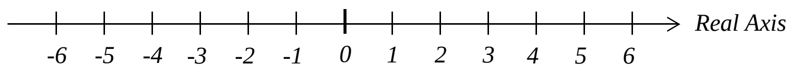


Figure 1.1. Representation of Real Numbers

In our subsequent discussion, we will only be concerned with real numbers and thus the word *real* will not be used further.

1.3 Addition and Subtraction

The following rules apply for the addition of numbers.

Rule 1: To add numbers with the same sign, we add the absolute values of these numbers and place the common sign (+ or –) in front of the result (sum). We can omit the plus sign in the result if positive. We must not omit the minus sign if the result is negative.

Example 1.3

Perform the addition

$$7 + 16 + 0.5$$

Solution:

The plus sign between the given numbers indicates addition of three positive numbers whose sign is positive and it is omitted. However, we can enclose these numbers in parentheses just to emphasize that the numbers are positive. Addition of the absolute values of these numbers yield a

* *The reader may have heard the expression "**imaginary numbers**". The square root of minus 1, i.e, $\sqrt{-1}$, is an example of an imaginary number; it does not fit anywhere in the real axis of numbers. We will not be concerned with these numbers in this text. There is a brief discussion in Appendix A in conjunction with MATLAB.*

** *Only whole numbers are shown on the real axis of Figure 1.1. However, it is understood that within each division, there are numbers such as 1.5, –2.75 etc.*

sum of 23.5, and this also represents an absolute value. We should remember that the absolute value of a number is that just that number without regard to being positive or negative. Now, since all three numbers are positive, the sum is $+23.5$ or simply 23.5 as shown below. The final result, 23.5, does not represent an absolute number; it is a positive number whose sign has been omitted, as it is customary. Thus,

$$7 + 16 + 0.5 = (+7) + (+16) + (+0.5) \Rightarrow |7| + |16| + |0.5| = |23.5| \Rightarrow (+23.5) = 23.5$$

where the symbol $\Rightarrow$ means conversion from signed numbers to absolute values and vice versa. Of course, these steps will be unnecessary after one becomes familiar with the rules.

Example 1.4

Perform the addition

$$(-6) + (-45)$$

Solution:

Here, we are asked to add two negative numbers as indicated by the addition sign between them. Addition of the absolute values yields 51 and since both numbers are negative, we place the minus sign in front of the result. Therefore, the sum of these numbers is (–51) or –51. The minus sign cannot be omitted. Thus,

$$(-6) + (-45) \Rightarrow |6| + |45| = |51| \Rightarrow (-51) = -51$$

Example 1.5

Perform the addition

$$(-1.25) + (-0.75) + (-3)$$

Solution:

In this example, we are asked to add three negative numbers. The sum of the absolute values is 5. Since all given numbers are negative, we place the minus sign in front of the sum. The result then is (-5) or simply -5. The negative sign cannot be omitted. Thus,

$$(-1.25) + (-0.75) + (-3) \Rightarrow |1.25| + |0.75| + |3| = |5| \Rightarrow (-5) = -5$$

Consider the subtraction of number B from number A, that is, $A-B$. The number A is called the *minuend* and the number B is called the *subtrahend*. The result of the subtraction is called the *difference*. These definitions are illustrated with Examples 1.6 and 1.7 below.

Example 1.6

Draw a rough sketch to indicate the minuend, subtrahend, and difference for the subtraction operation of $735-592$.

Solution:

The minuend, subtrahend, and difference are shown on the sketch below. The procedure for finding the difference will be explained in Rule 3 below.

$$735 - 592 = 143$$

Difference
Subtrahend
Minuend

Example 1.7

Draw a rough sketch to indicate the minuend, subtrahend, and difference for the subtraction operation of $248 - 857$.

Solution:

The minuend, subtrahend, and difference are shown on the sketch below. The procedure for finding the difference is our next topic.

$$248 - 857 = -609$$

Difference
Subtrahend
Minuend

Rule 2: ***To add two numbers with different signs, we subtract the number with the smaller absolute value from the number with the larger absolute value, and we place the sign of the larger number in front of the result (sum).***

Example 1.8

Perform the addition

$$37 + (-15)$$

Solution:

The number with the smaller absolute value is 15; therefore, we subtract this from the absolute value of the larger number, 37, and we place the plus sign in front of the difference as shown below because the larger number is positive. Since the result is a positive number, we omit the plus sign.

$$37 + (-15) = (+37) + (-15) \Rightarrow |37| - |15| = |22| \Rightarrow (+22) = 22$$

Example 1.9

Perform the addition

$$(-16) + 7$$

Solution:

The number with the smaller absolute value is *7*; therefore, we subtract this from the absolute value of the larger number which is *16*, and we place the minus sign in front of the difference as shown below.

$$(-16) + 7 = (-16) + (+7) \Rightarrow |16| - |7| = |9| \Rightarrow (-9) = -9$$

Rule 3: To subtract one number (the subtrahend) from another number (the minuend) we change the sign (we replace + with – or – with +) of the subtrahend, and we perform addition instead of subtraction.

Example 1.10

Perform the subtraction

$$39 - 25$$

Solution:

For this example, both the minuend and subtrahend are positive numbers. The minus sign between them indicates subtraction. Therefore, we enclose them in parentheses with the positive (+) sign, and we change the sign of the subtrahend from plus to minus, while at the same time, we change the subtraction operation to addition. Next, we need to add these numbers, one of which is positive and the other negative. For this addition we follow Rule 2, that is, we subtract the number with the smaller absolute number from the larger. The steps are shown below.

$$39 - 25 = (+39) - (+25) = (+39) + (-25) \Rightarrow |39| - |25| = |14| \Rightarrow +14 = 14$$

Example 1.11

Perform the subtraction

$$53 - (-18)$$

Solution:

Here, the minuend is positive, the subtrahend is negative, and the minus sign between then indicates subtraction. Therefore, we change the sign of the subtrahend from minus to plus and at the same time we change the subtraction operation to addition. Next, we need to add these two positive numbers, and for this addition we follow Rule 1, that is, we add the absolute values and we place the plus sign in front of the result. The steps are shown below.

$$53 - (-18) = (+53) - (-18) = (+53) + (+18) \Rightarrow |53| + |18| = |71| \Rightarrow +71 = 71$$

Again, these steps indicate the "train of thought" and the reader is not expected to write down all these steps when performing arithmetic operations.

Example 1.12

Perform the subtraction

$$(-86) - 37$$

Solution:

Here, the minuend is negative, the subtrahend is positive, and the minus sign between them indicates subtraction. Therefore, we enclose 37 in parentheses with the positive (+) sign, then, change the sign of it from plus to minus, and at the same time we change the subtraction operation to addition. Next, we must add these two negative numbers, and for this addition we follow Rule 1, that is, we add the absolute values and we place the minus sign in front of the result. The steps are shown below.

$$(-86) - 37 = (-86) - (+37) = (-86) + (-37) \Rightarrow |86| + |37| = |123| \Rightarrow -123$$

Example 1.13

Perform the subtraction

$$(-75) - (-125)$$

Solution:

Here, both the minuend and subtrahend are negative, and the minus sign between them indicates subtraction. Therefore, we change the sign of the subtrahend from minus to plus, and at the same time we change the subtraction operation to addition. Next, we must add these numbers which now have different signs. For this addition we follow Rule 2, that is, we subtract the number with the smaller absolute number from the larger, and we place the sign of the larger number in front of the result. The steps are shown below.

$$(-75) - (-125) = (-75) + (+125) \Rightarrow |125| - |75| = |50| \Rightarrow +50 = 50$$

In general, let X be any number; then, the following relations apply for the possible combinations of addition and subtraction of positive and negative numbers.

$$\begin{aligned} +(+X) &= +X = X \\ +(-X) &= -X \\ -(+X) &= -X \\ -(-X) &= X \end{aligned} \qquad (1.1)$$

1.4 Multiplication and Division

Note 1.1

The multiplication (×) and division (÷) signs do not interfere with the plus and minus signs of positive and negative numbers; therefore, for the examples which follow, we will omit the steps involving absolute values.

Note 1.2

Multiplication is repeated addition, for instance, $5 \times 3 = 5 + 5 + 5 = 15$. One of the two numbers involved in multiplication is the *multiplier,* and the other is the *multiplicand.* The dictionary defines multiplier the number by which another number is multiplied. The multiplicand is defined as the number that is or is to be multiplied by another. Thus, in 5×3, the multiplier is 5 and the multiplicand is 3. But since we can change the order of multiplication as $5 \times 3 = 3 \times 5$, either number can be the multiplicand or the multiplier. In this text, we will refer to the first number as the multiplier and the second as the multiplicand. The result of the multiplication is called the *product*.

The following rules apply for multiplication and division of numbers:

Rule 4: When two numbers with the same sign are multiplied, the product will be positive (+). If the numbers have different signs, the product will be negative (–).

Example 1.14

Perform the multiplication

$$39 \times 25$$

Solution:

For this example, both the multiplier and multiplicand are positive. Since they have the same sign, the product will be positive. The steps are shown below.

$$39 \times 25 = (+39) \times (+25) = (+975) = 975$$

Note 1.3

The reader can use a hand calculator to obtain the result. The intent here is to illustrate how the sign of the result is obtained. The same is true for the examples that follow.

Example 1.15

Perform the multiplication

$$53 \times (-18)$$

Solution:

For this example, the multiplier is positive and the multiplicand is negative. Since they have different signs, in accordance with Rule 4 the product will be negative. The steps are shown below.

$$53 \times (-18) = (+53) \times (-18) = (-954) = -954$$

Example 1.16

Perform the multiplication

$$(-86) \times 37$$

Solution:

Here, the multiplier is negative and the multiplicand is positive. Since they have different signs, in accordance with Rule 4, the product will be negative. The steps are shown below.

$$(-86) \times 37 = (-86) \times (+37) = (-3182) = -3182$$

Example 1.17

Perform the multiplication

$$(-75) \times (-125)$$

Solution:

For this example, both the multiplier and multiplicand are negative. Since they have the same sign, in accordance with Rule 4, the product will be positive. The steps are shown below.

$$(-75) \times (-125) = (+9375) = 9375$$

Note 1.4

A *rational number* is a number that can be expressed as an integer or a quotient of integers, excluding zero as a denominator. In division, the number that is to be divided by another is called the *dividend*. For instance, the number $\frac{27}{3}$ is a rational number and the dividend is 27. The dividend is also referred to as the *numerator*. The number by which the dividend is to be divided, is called the *divisor*. For instance, in the division operation above, the divisor is 3. The divisor is also called the *denominator*. The number obtained by dividing one quantity by another is the *quotient*. Thus, in $\frac{27}{3} = 9$, the quotient is 9.

Rule 5: When two numbers with the same sign are divided, the quotient will be positive (+). If the numbers have different signs, the quotient will be negative (–).

Example 1.18

Perform the division

$$\frac{125}{5}$$

Solution:

Here, both the dividend and divisor are positive. Since they have the same sign, in accordance with Rule 5 the quotient will be positive. The steps are shown below.

$$\frac{125}{5} = \frac{(+125)}{(+5)} = (+25) = 25$$

Example 1.19

Perform the division

$$\frac{-750}{10}$$

Solution:

Here, the dividend is negative and the divisor is positive. Since they have different signs, in accordance with Rule 5 the quotient will be negative. The steps are shown below.

$$\frac{-750}{10} = \frac{(-750)}{(+10)} = (-75) = -75$$

Example 1.20

Perform the division

$$\frac{1500}{-75}$$

Solution:

Here, the dividend is positive and the divisor is negative. Since they have different signs, in accordance with Rule 5 the quotient will be negative. The steps are shown below.

$$\frac{1500}{-75} = \frac{(+1500)}{(-75)} = (-20) = -20$$

Example 1.21

Perform the division

$$\frac{-450}{-15}$$

Solution:

Here, both the dividend and divisor are negative. Since they have the same sign, in accordance with Rule 5 the quotient will be positive. The steps are shown below.

$$\frac{-450}{-15} = \frac{(-450)}{(-15)} = (+30) = 30$$

1.5 Integer, Fractional, and Mixed Numbers

A *decimal point* is the dot in a number that is written in decimal form. Its purpose is to indicate where values change from positive to negative powers of 10. Positive and negative powers will be discussed in Section 1.8. For instance, the number 5.3 is written in decimal form, and the dot between 5 and 3 is referred to as the decimal point.

Every number has a decimal point although it may not be shown. A number is classified as an *integer*, *fractional* or *mixed* depending on the position of the decimal point in that number.

An *integer* number is a whole number and it is understood that its decimal point is positioned after the last (rightmost) digit although not shown. For instance,

$$7 = 7. \qquad -305 = -305. \qquad 14108 = 14108.$$

The numbers we have used in all examples thus far are integer numbers.

A *fractional* number, positive or negative, is always a number less than one. It can be expressed in a rational form such as $\frac{3}{4}$ or in *decimal point form* such as 0.75. When written in decimal point form, the decimal point appears in front of the first (leftmost) digit. For instance,

$$\frac{1}{2} = .5 = 0.5 \qquad\qquad \frac{-1}{4} = -.25 = -0.25$$

where $\frac{1}{2}$ and $\frac{-1}{4}$ are fractional numbers expressed in rational form, and 0.5 and -0.25 are fractional numbers expressed in decimal point form.

Note 1.5

When writing fractional numbers in decimal form, it is highly recommended that a 0 (zero) is written in front of the decimal point. This reduces the possibility of an erroneous reading if the decimal point goes unnoticed. The presence of a zero in front of the decimal point alerts the reader that a decimal point may follow the zero. We will follow this practice throughout this text.

A *mixed number* consists of an integer and a fractional number. For instance, the number 409.0875 consists of the integer part 409 and the fractional part 0.0875 as shown below.

$$409.0875 = 409. + 0.0875$$

Fractional
Integer
Mixed

Rule 6: Any number (integer, fractional or mixed) may be written in a rational form where the numerator (dividend) is the number itself and the denominator (divisor) is 1.

For instance, the numbers 13, 385, 2.75, and $22/7$ may be written as

$$13 = \frac{13}{1} \qquad 385 = \frac{385}{1} \qquad 2.75 = \frac{2.75}{1} \qquad \frac{22}{7} = \frac{\frac{22}{7}}{1} \qquad -0.25 = -\frac{0.25}{1}$$

1.6 Reciprocals of Numbers

The *reciprocal* of an integer number is a fraction with numerator 1 and denominator that number itself. Alternately, since any number can be considered as a fraction with denominator 1, the reciprocal of a fraction is another fraction with the numerator and denominator reversed. Consequently, the product (multiplication) of any number by its reciprocal is 1.

Example 1.22

Find the reciprocals of

a. 3 b. $\frac{1}{4}$ c. -3.875 d. $\frac{-1}{2}$

Solution:

a.
$$\text{The reciprocal of } 3 \text{ is } \frac{1}{3}$$

b.
$$\text{The reciprocal of } \frac{1}{4} \text{ is } \frac{4}{1} \text{ or } 4$$

c.
$$\text{The reciprocal of } -3.875 \text{ is } \frac{1}{(-3.875)} = -0.2581$$

d.
$$\text{The reciprocal of } \frac{-1}{2} \text{ is } \frac{2}{-1}$$

Note 1.6

When either the sign of the numerator or the denominator (but not both) is negative, it is customary to place the minus sign in front of the bar that separates them. For instance,

$$\frac{-5}{8} = -\frac{5}{8} \qquad \frac{4}{-11} = -\frac{4}{11}$$

We must not forget that when the sign of both the numerator and denominator is minus, the result is a positive number in accordance with *Rule 5*.

Rule 7: ***A rational number in which the numerator and denominator are the same number is always equal to 1 with the proper sign.***

For instance,

$$\frac{7}{7} = 1 \qquad \frac{-12}{12} = -\frac{12}{12} = -1$$

Rule 8: ***As a consequence of Rule 7, the value of a rational number is not changed if both numerator and denominator are multiplied by the same number.***

1.7 Arithmetic Operations with Fractional Numbers

The following rules apply to addition, subtraction, multiplication and division with fractional numbers. As stated in Rule 8, the value of a rational number is not changed if both the numerator and denominator are multiplied by the same number.

Rule 9: ***To add two or more fractional numbers in rational form, we first express the numbers in a common denominator; then, we add the numerators to obtain the numerator of the sum. The denominator of the sum is the common denominator. If the numbers are in decimal point form, we first align the given numbers with the decimal point, and we add the numbers as it is done in normal addition.***

Example 1.23

Perform the addition

$$\frac{1}{2} + \frac{1}{4}$$

Solution:

Before we add these numbers, we must express them in a common (same) denominator form. Here $1/2$ is the same as $2/4$* and this and the number $1/4$ have the same (common) denominator. Then, by Rule 9,

* *Recall that, in accordance with Rule 8, we can multiply both numerator and denominator of $1/2$ by 2 to get $2/4$.*

$$\frac{1}{2}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4} = \frac{3}{4}$$

Note 1.7

In an expression such as the above, the individual numbers $1/2$ and $1/4$ are referred to as the *terms* of the expression. Of course, an expression may also have three or more terms.

Example 1.24

Perform the addition

$$\frac{1}{25}+\frac{1}{10}+\frac{1}{20} \tag{1.2}$$

Solution:

To perform this addition, we must first express the given numbers so that all have the same denominator. This can be done by first finding the *Least Common Multiple (LCM).* The LCM is the smallest quantity that is divisible by two or more given quantities without a remainder. For this example, the *LCM* is *100* which, when divided by *25*, *10* and *20* yields *4, 10* and *5* respectively. We can verify that the *LCM* is *100* with *Microsoft Excel** as follows.

Start Excel. The spreadsheet appears with a rectangle in cell A1. The cell which is surrounded by this rectangle is the *selected* cell. Thus, when Excel is first invoked (started), the selected cell is A1. You can select any other cell by moving the thick hollow white cross pointer to the desired cell and click the mouse. Select cell A3. Next, move the mouse towards the top of the screen where several *icons* (symbols and pictures) appear. This is the *Standard Toolbar.* Click on f_x. The *Function Category* menu appears. Select *Math & Trig*. Scroll down the function name list to locate LCM and click on it. In Number 1 box type *25*, in Number 2 type *10,* and in Number 3 type *20.* Click on OK and observe that Excel displays *100.* This is the LCM for this example.

Next, we need to express each of the terms of (1.2) with the common denominator *100*. Of course, we must multiply the numerators of each term with the appropriate number so that numerical value of each term will not change. The new numerator values are found by multiplying each numerator by the quotient obtained by dividing 100 by each of the denominators of (1.2).

Thus, we find the new numerator of $\frac{1}{25}$ by first dividing *100* by *25* and this division yields *4*. Then, we multiply this number by the numerator *1* and we get $1 \times 4 = 4$. Therefore, the number $\frac{1}{25}$ is

* *No proficiency with Excel is required. However, the reader must have a Personal Computer (PC) or McIntosh, or access to one in which Excel has been installed. Otherwise, he may use a hand calculator which has the LCM feature.*

now expressed as $\frac{4}{100}$. Likewise, we express $\frac{1}{10}$ as $\frac{10}{100}$ and $\frac{1}{20}$ as $\frac{5}{100}$. In other words, we have multiplied the numerator and denominator of each term by the same number so that their values will not change. Then, by Rule 9 we have:

$$\frac{1}{25}+\frac{1}{10}+\frac{1}{20} = \frac{1\times 4}{25\times 4}+\frac{1\times 10}{10\times 10}+\frac{1\times 5}{20\times 5} = \frac{4}{100}+\frac{10}{100}+\frac{5}{100} = \frac{19}{100}$$

We can verify the answer directly with Excel as follows:

Start with a blank worksheet. Click on the letter *A* of Column A and observe that the entire column is now highlighted, that is, the background on this column has changed from white to black. Click on ***Format*** on the Menu bar (top of the screen) and on the drop menu click on ***Cells***. The Format Cells menu appears. Under the Category column select ***Fraction*** and on the right side, under ***Type***, select ***Up to three digits***. Click on OK.

In cell A1 type 1/25, in A2 1/10 and in A3 1/20. Observe that these numbers appear on the spreadsheet as entered. After the last entry and pressing the <*enter*> key, the selected cell is A4. Now, click on the Σ (sum) symbol on the *Standard Toolbar.* Excel displays =SUM(A1:A3) in A4. Excel is asking us if we want to add the contents of A1 through A3 and display the answer in A4. Of course, this is what we want and we confirm it by pressing the <*enter*> key. Excel then displays the answer in A4 which is 19/100.

Note 1.8

For brevity, when using Excel in our subsequent discussion, we will omit the word cell and the key <*enter*>. Thus, B3, C11 etc., will be understood to be cell B3, cell C11 etc. After an entry in a cell has been made, it will be understood that the <*enter*> key was pressed. Also, we will denote sequential selections with the > (greater than) symbol. Thus, in the example above, our preference to display the numbers in rational form was accomplished by the sequential steps ***Format>Cells>Fraction>Up to three digits>OK***.

In Examples 1.23 and 1.24 above, finding the common denominator was an easy task even without using Excel. However, in other cases, it may be a tedious process. Consider the following example.

Example 1.25

Perform the following addition and express the answer in rational form.

$$\frac{3}{4}+\frac{2}{5}+\frac{4}{7}+\frac{11}{12}+\frac{7}{16}+\frac{19}{22} \tag{1.3}$$

Solution:

To find the *LCM* here without the use of a hand calculator or a PC, is a formidable task. Therefore, we will use Excel as we did in the previous example. Recall that *LCM* produces the common denominator of two or more numbers. The common multiplier for the numbers *4, 5, 7, 12, 16* and

22 is found as follows:

Start with a blank Excel worksheet. Following the same procedure as in Example 1.24, click on f_x *in the Standard Toolbar***.** The *Function Category* menu appears. Select *Math & Trig*. Scroll down the *function name* list to locate LCM and click on it. In Number 1 box enter *4, 5, 7, 12, 16, 22* separated by commas, and observe that Excel displays *18480.* This is the *LCM* for this example.

Now, we need to express each of the terms of (1.3) with the common denominator *18480*. Of course, we must multiply the numerators of each term with the appropriate number so that numerical value of each term will not change. The new numerator value of each term is found by multiplying the present numerator by the quotient that is obtained with the division of *18480* by its respective denominator. Thus, we find the new numerator of $\frac{3}{4}$ by first dividing *18480* by *4;* this division yields *4620*. Then, we multiply this number by the present numerator of this term, that is, *3* and we get $3 \times 4620 = 13860$. Therefore, the number $\frac{3}{4}$ is expressed as $\frac{13860}{18480}$. In other words, we multiplied both numerator and denominator of $\frac{3}{4}$ by *4620*. Likewise, to find the new numerator of the term $\frac{2}{5}$, we first divide *18480* by *5* to get *3696*. Then, multiplication of this number by *2* yields the new numerator of the second term which is $2 \times 3696 = 7392$, and therefore we express $\frac{2}{5}$ as $\frac{7392}{18480}$. Following the same procedure, we express $\frac{4}{7}$ as $\frac{10560}{18480}$, $\frac{11}{12}$ as $\frac{16940}{18480}$, $\frac{7}{16}$ as $\frac{8085}{18480}$ and $\frac{19}{22}$ as $\frac{15960}{18480}$. Finally, we find the sum of the given numbers in (1.3) as

$$
\begin{aligned}
&\frac{3}{4}+\frac{2}{5}+\frac{4}{7}+\frac{11}{12}+\frac{7}{16}+\frac{19}{22} \\
&= \frac{3\times 4620}{4\times 4620}+\frac{2\times 3696}{5\times 3696}+\frac{4\times 2640}{7\times 2640}+\frac{11\times 1540}{12\times 1540}+\frac{7\times 1155}{16\times 1155}+\frac{19\times 840}{22\times 840} \\
&= \frac{13860}{18480}+\frac{7392}{18480}+\frac{10560}{18480}+\frac{16940}{18480}+\frac{8085}{18480}+\frac{15960}{18480} = \frac{72797}{18480}
\end{aligned}
$$

This example required a considerable amount of work. This was the procedure one would follow before the advent of scientific calculators and personal computers. Let us now verify the result with Excel directly, that is, without first computing the *LCM*.

Start with a blank worksheet. Click on the letter *A* of Column A and observe that the entire column is now highlighted, that is, the background on this column has changed from white to black. Click on ***Format>Cells>Fraction>Up to three digits>OK***.

In A1 through A6*,* type 3/4, 2/5, 4/7, 11/12/ 7/16 and 19/22 respectively. Observe that these numbers appear on the spreadsheet as entered. After the last entry and pressing the <*enter*> key, the selected cell is *A7*. Now, click on the Σ (sum) symbol on the *Standard Toolbar.* Excel displays =SUM(A1:A6) in A7, and is asking us if we want to add the contents of *A1* through A6 and display the answer in A7. Of course, this is what we want and we confirm it by pressing the <*enter*> key. Excel now displays the answer in A4 which is 3881/938. This value is an approximation since Excel can only display rational numbers up to 3 digits. The exact value is $\frac{72797}{18480}$ as found above. We will learn how to compute the percent error in Section 1.14.

Note 1.9

The reader, at this time, may ask why should one waste his time trying to find the solution to a problem when it can easily be found with a hand calculator or a PC. There is nothing wrong with this thought provided that we can predict the *approximate* answer beforehand, or we can determine whether the answer displayed is reasonably correct. We must remember that a PC is a high speed idiot and will do exactly whatever we will ask it to do. So, if we accidentally enter the wrong numbers, the answer we will get will definitely be wrong and unless we challenge it, we will accept it as the correct answer. Likewise, with hand calculators; they display the wrong numbers when inaccurate numbers are entered, and also when the batteries are running low.

Example 1.26

Perform the addition

$$\frac{1}{15}+\left(-\frac{1}{10}\right)$$

Solution:

The LCM for this example is 90. We can verify the answer with Excel as we did in the previous examples. This is left as an exercise for the reader. Then,

$$\frac{1}{15}+\left(-\frac{1}{10}\right) = \frac{1\times 6}{15\times 6}+\left(-\frac{1\times 9}{10\times 9}\right) = \frac{6}{90}+\left(-\frac{9}{90}\right) = -\frac{3}{90} = -\frac{1}{30}$$

Note 1.10

In the last step above, we divided both the numerator and denominator of $-\frac{3}{90}$ by 3, or we can say that we multiplied each of these by $\frac{1}{3}$.

Example 1.27

Perform the addition

$$-\frac{5}{12}+\frac{11}{24}$$

Solution:

The LCM for this example is *24*. Then,

$$-\frac{5}{12}+\frac{11}{24} = \left(-\frac{5\times 2}{12\times 2}\right)+\frac{11\times 1}{24\times 1} = \left(-\frac{10}{24}\right)+\frac{11}{24} = \frac{1}{24}$$

Example 1.28

Perform the addition

$$0.75 + 0.005$$

Solution:

These numbers are given in decimal point (not rational) form. We can express them in rational form as $\frac{75}{100}$ and $\frac{5}{1000}$, find the LCM, which, in this case, is *1000,* and add them as we did in Examples 1.23 through 1.27. However, it is easier to add them directly as they appear by aligning them with the decimal point, then add them as it is done in normal addition. This is shown below.

$$0.75 + 0.005 \longrightarrow \begin{array}{r} 0.750 \\ +\;0.005 \\ \hline 0.755 \end{array}$$

We will do this simple example with Excel just to become more familiar with spreadsheets.

Start with a blank worksheet or erase the contents of a previous worksheet. We erase the spreadsheet contents by surrounding all entries with a big rectangle that encloses all previous entries. When this is done properly, the selected cells within the rectangle will be highlighted. Press the *Delete* key on the keyboard and observe that all cells are now blank. Click on the letter *A* of Column A and observe that the entire column is now highlighted. Click on ***Format>Cells>***(tab)***Number>***(selection)***Number*** and the decimal places box click on the up or down arrow until the number *3* appears. Click on OK. On A1 type *0.750* and on A2 *0.005*. The selected cell is now A3. Next, click on the Σ (sum) symbol on the Standard Toolbar. Excel displays =SUM(A1:A2) in A3, and Excel is asking us if we want to add the contents of A1 and A2 in display the answer in *A3*. This is what we want and we confirm it by pressing the *<enter>* key. Excel then displays the answer in A3 which is *0.755*.

Using Excel for this example, it was not necessary that the numbers are all displayed with the same number of decimal places. Excel will produce the correct answer if the numbers typed-in do not have the same number of decimal places. We chose to display them as we did just to become more familiar with Excel's format feature.

Rule 10: ***To subtract a number given in rational form from another number of the same form, we first express the numbers in a common denominator; then, we subtract the numerator of the subtrahend from the numerator of the minuend to obtain the numerator of the difference. The denominator of the difference is the common denominator. If the numbers are in decimal point form, we first align the given numbers with the decimal point; then, we subtract the subtrahend from the minuend as it is done in normal subtraction.***

Example 1.29

Perform the following subtractions.

a. $\frac{1}{10}-\frac{1}{20}$ b. $\frac{7}{25}-\left(-\frac{1}{10}\right)$ c. $\left(-\frac{5}{16}\right)-\frac{1}{24}$ d. $0.75-0.005$

Solution:

For (a), (b) and (c) we follow the same steps as we did in Examples 1.23 through 1.27 except that here we perform subtraction instead of addition. Thus,

a.
$$\frac{1}{10}-\frac{1}{20} = \frac{2}{20}-\frac{1}{20} = \frac{1}{20}$$

b.
$$\frac{7}{25}-\left(-\frac{1}{10}\right) = \frac{14}{50}+\frac{5}{50} = \frac{19}{50}$$

c.
$$\left(-\frac{5}{16}\right)-\frac{1}{24} = \left(-\frac{15}{48}\right)-\frac{2}{48} = \left(-\frac{17}{48}\right)$$

d. We follow the same steps as Example in 1.28, except that we perform subtraction instead of addition. Then,

$$0.75-0.005 \longrightarrow \begin{array}{r} 0.750\ - \\ \underline{0.005} \\ 0.745 \end{array}$$

Rule 11: ***To multiply a number given in a rational form by another number of the same form, we multiply the numerators to obtain the numerator of the product. Then, we multiply the denominators to obtain the denominator of the product. If the numbers are in decimal point form, we multiply the given numbers as is done in normal multiplication, and we place the decimal point at the position that is equal to the sum of the decimal places of the given numbers.***

Example 1.30

Perform the following multiplications.

a. $\frac{1}{12} \times \frac{1}{10}$ b. $\frac{3}{25} \times \left(-\frac{1}{10}\right)$ c. $\left(-\frac{23}{8}\right) \times \frac{1}{12}$ d $.0.75 \times 0.005$

Solution:

For each of (a), (b) and (c) we multiply the numerators to obtain the numerator of the product, then multiply the denominators to get the denominator of the product in accordance with Rule 11. Then,

a. $$\frac{1}{12} \times \frac{1}{10} = \frac{1 \times 1}{12 \times 10} = \frac{1}{120}$$

b. $$\frac{3}{25} \times \left(-\frac{1}{10}\right) = -\frac{3 \times 1}{25 \times 10} = -\frac{3}{250}$$

c. $$\left(-\frac{23}{8}\right) \times \frac{1}{12} = -\frac{23 \times 1}{8 \times 12} = -\frac{23}{96}$$

d. We first perform the multiplication $75 \times 5 = 375$. Next, we place the decimal point at the position that is equal to the *sum* of the decimal places of the given numbers starting from the rightmost digit of the number we found, i.e., *375*, and proceeding to the left. For this example, the multiplier has two decimal places and the multiplicand three; thus, their sum is five. The product is as shown below.

$$0.75 \times 0.005 \longrightarrow \begin{array}{r} 0.75 \times \\ 0.005 \\ \hline 0.00375 \end{array}$$

Rule 12: To divide a number (dividend) given in a rational form, by another number (divisor) of the same form, we multiply the numerator of the dividend by the denominator of the divisor to obtain the numerator of the quotient. Then, we multiply the denominator of the dividend by the numerator of the divisor to obtain the denominator of the quotient. If the numbers are in decimal point form, we first express them in rational form and convert to integers by multiplying both numerator and denominator by the same smallest possible number that will make them integer numbers; then, we perform division as stated in Rule 5.

Example 1.31

Perform the following divisions.

a. $\frac{1}{12} \div \frac{1}{10}$ b. $\frac{13}{12} \div \left(-\frac{1}{10}\right)$ c. $\left(-\frac{15}{8}\right) \div \frac{5}{12}$ d. $0.75 \div 0.005$

Solution:

For each of (a), (b) and (c) we multiply the numerator of the dividend by the denominator of the divisor to obtain the numerator of the quotient; then we multiply the denominator of the dividend by the numerator of the divisor to get the denominator of the quotient in accordance with Rule 12. Then,

a.
$$\frac{1}{12} \div \frac{1}{10} = \frac{1 \times 10}{12 \times 1} = \frac{10}{12} = \frac{5}{6}$$

b.
$$\frac{13}{12} \div \left(-\frac{1}{10}\right) = -\frac{13 \times 10}{12 \times 1} = -\frac{130}{12} = -\frac{65}{6}$$

c.
$$\left(-\frac{15}{8}\right) \div \frac{5}{12} = -\frac{15 \times 12}{8 \times 5} = -\frac{180}{40} = -\frac{9}{2}$$

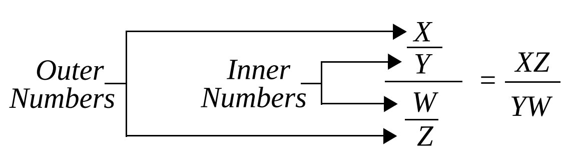
d. We first express the given numbers in rational form and we multiply both numerator and denominator by *1000* to convert them to integers. The division is carried in accordance with Rule 5. Then,

d. $$0.75 \div 0.005 \longrightarrow \frac{0.75 \times 1000}{0.005 \times 1000} = \frac{750}{5} = 150$$

Often, the division of two numbers in ratio form is expressed in complex fraction form. A *complex fraction* is a fraction whose numerator or denominator or both are fractions. The general form of a complex fraction is

$$\frac{\frac{X}{Y}}{\frac{W}{Z}}$$

Any complex fraction can be simplified by application of Rule 12 above twice. Alternately, any complex fraction can be replaced by a simple fraction whose numerator is the product of the outer numbers X and Z, and whose denominator is the product of the inner numbers Y and W, as shown in the sketch below.



Example 1.32

Simplify the following complex fractions.

a. $\dfrac{\frac{3}{5}}{\frac{4}{7}}$ b. $\dfrac{5}{\frac{3}{4}}$ c. $\dfrac{-\frac{1}{4}}{0.4}$

Solution:

a. We multiply the outer numbers, i.e., 3×7 to obtain the numerator and the inner numbers, i.e., 5×4 to obtain the denominator. Then,

$$\frac{\frac{3}{5}}{\frac{4}{7}} = \frac{3 \times 7}{5 \times 4} = \frac{21}{20}$$

b. The numerator 5 of the given complex fraction can be expressed in rational form as $5/1$. Then, performing the same steps as in (a), we get

$$\frac{5}{\frac{3}{4}} = \frac{\frac{5}{1}}{\frac{3}{4}} = \frac{5 \times 4}{1 \times 3} = \frac{20}{3}$$

c. The denominator 0.4 is first expressed as $\frac{4}{10}$. Then,

$$\frac{-\frac{1}{4}}{0.4} = \frac{-\frac{1}{4}}{\frac{4}{10}} = -\frac{1 \times 10}{3 \times 4} = -\frac{10}{12} = -\frac{5}{6}$$

1.8 Exponents

In the mathematical expression $\boldsymbol{a^n}$, $\boldsymbol{a}$ is the *base*, $\boldsymbol{n}$ is the *exponent* or *power*, and $\boldsymbol{a^n}$ is said to be a *number in exponential form.* The exponent indicates the number of times the base appears in a string of multiplication operations as illustrated by the following example.

Example 1.33

Express the following numbers in exponential form.

a. $10 \times 10 \times 10 \times 10$ b. $5 \times 5 \times 5$ c. 3×3 d. $7 \times 7 \times 7 \times 8 \times 8$ e. $100, 000, 000$

Solution:

a. The number (base) 10 appears 4 times and thus the exponent is 4. Then,

$$10 \times 10 \times 10 \times 10 = 10^4 \quad \text{(Read as } 10 \text{ to the } 4th \text{ power)}$$

10 is the base and *4* is the exponent (or power).

b. The number (base) 5 appears 3 times and thus the exponent is 3. Then,

$$5 \times 5 \times 5 = 5^3 \quad \text{(Read as 5 to the 3rd power or 5 cubed)}$$

5 is the base and *3* is the exponent.

c. The number (base) 3 appears 2 times and thus the exponent is 3. Then,

$$3 \times 3 = 3^2 \quad \text{(Read as 3 to the 2nd power or 3 squared)}$$

3 is the base and *2* is the exponent.

d.
$$7 \times 7 \times 7 \times 8 \times 8 = 7^3 \times 8^2 \quad \text{(Read as 7 cubed times 8 squared)}$$

7 and 8 are the bases and *3* and *2* are the exponents.

e. As we now know, $100 = 10 \times 10 = 10^2$, $1000 = 10 \times 10 \times 10 = 10^3$ and so on. We observe that the exponent is equal to the number of zeros. Conversely, as we found in (a), the exponent *4* represents the four zeros of the number $10,000$. Here, the number $100,000,000$ has 8 zeros and thus can be written in exponential form with base *10* and exponent 8, that is,

$$100,000,000 = 10^8$$

Rule 13: Any number written without an exponent is understood to have exponent 1.

Example 1.34

Determine the exponents of the following numbers.

a. 10 b. 3 c. -897 d. 0.06

Solution:

In accordance with Rule 13, the exponent of all these numbers is 1. Then,

a. 10^1 b. 3^1 c. -897^1 d. 0.06^1

Rule 14: Any number that has exponent 0 is equal to 1.

Example 1.35

Express the following numbers in their simplest form.

a. 10^0 b. 8^0 c. -1093^0 d. 0.005^0

Solution:

In accordance with Rule 14, these numbers are all equal to 1, that is,

a. $10^0 = 1$ b. $8^0 = 1$ c. $-1093^0 = 1$ d. $0.005^0 = 1$

Rule 15: Any number may be written in powers of 10 multiplied by the appropriate number of digits.

Example 1.36

Express the number *65,437* in powers of 10.

Solution:

$$\begin{aligned} 65437 &= 60,000 + 5,000 + 400 + 30 + 7 \\ &= 6 \times 10,000 + 5 \times 1,000 + 4 \times 100 + 3 \times 10 + 7 \\ &= 6 \times 10^4 + 5 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 7 \times 10^0 \end{aligned}$$

We observe that the exponents appear in descending order, that is, $4, 3, 2, 1, 0$. This is true for all numbers expressed as powers of 10.

Rule 16: Any number can be replaced by its reciprocal if the sign of its exponent is changed. Thus,

$$\frac{1}{X} = X^{-1} \qquad \text{and} \qquad Y = \frac{1}{Y^{-1}}$$

Example 1.37

Express the following numbers in exponential form.

a. 0.1 b. 0.01 c. -0.001 d. 0.5 e. $\frac{1}{0.1}$ f. $\frac{1}{0.01}$ g. $-\frac{1}{0.001}$ h. $\frac{1}{0.5}$

Solution:

The given numbers can be expressed in exponential form by first expressing them in rational form and then applying Rules 13 through 16. Thus,

a. $$0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$$

b. $$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

c. $$-0.001 = -\frac{1}{1000} = -\frac{1}{10^3} = -10^{-3}$$

d. $$0.5 = \frac{1}{2} = \frac{1}{2^1} = 2^{-1}$$

e. $$\frac{1}{0.1} = \frac{1}{10^{-1}} = 10^1 = 10$$

f. $$\frac{1}{0.01} = \frac{1}{10^{-2}} = 10^2 = 100$$

g. $$-\frac{1}{0.001} = -\frac{1}{10^{-3}} = -10^3 = -1000$$

h. $$\frac{1}{0.5} = \frac{1}{2^{-1}} = 2^1 = 2$$

For (e), (f), (g) and (h) we have used the results of (a), (b), (c) and (d) respectively.

1.9 Scientific Notation

In any number, the digit immediately to the left of the decimal point occupies the *units* position, the second digit occupies the *tens* position, the third the *hundreds* position, the fourth the *thousands* position and so on. The leftmost digit is the *most significant digit (msd)* and the rightmost is the *least significant digit (lsd)*. Thus, in the number *7469.32*, the msd is *7* and occupies the thousands position, *9* occupies the units position and *2* is the lsd.

Rule 17: ***A number is said to be written in scientific notation when the decimal point appears immediately to the right of the msd.***

Therefore, any single digit number such as *1* or *8*, or a two-digit number with a decimal point between the digits such as *6.5* or *1.9* is already in scientific notation. Other numbers can be written in scientific notation using the following rules:

Rule 18: ***To write an integer or a mixed number in scientific notation, we move the decimal point to the left until we reach the msd. We place it immediately to the right of the msd. Then, we multiply the new number by 10 raised to the power that is equal to the number of positions the decimal point was moved to the left.***

Example 1.38

Write the following numbers in scientific notation.

a. *29* b. *765* c. *203982* d. *–345.67* e. *98765.43*

Solution:

In accordance with Rule 18, for (a) we move the decimal point to the left by one position and we multiply by 10 with exponent 1. Likewise, for (b) we move the decimal point to the left by two positions and we multiply by 10 with exponent 2. We follow the same procedure for (c), (d) and (e). Then,

a. $$29 = 29. = 2.9 \times 10^1 = 2.9 \times 10$$

b. $$765 = 765. = 7.65 \times 10^2$$

c. $$203982 = 203982. = 2.03982 \times 10^5$$

d. $$-345.67 = -3.4567 \times 10^2$$

e. $$98765.43 = 9.876543 \times 10^4$$

Note 1.11

Scientific calculators and Excel display numbers in scientific notation using the letter E followed by a number that represents the exponent. For instance, the number 98765.43 in scientific notation is displayed as 9.86543E+4. For practice, type in the number 98765.43 in any cell and highlight that cell. Then, perform the sequential steps ***Format>Cells>Scientific>6(***Decimal places***)>OK***. Observe that the number is displayed as 9.86543E+4.

Rule 19: To write a fractional number in scientific notation, we move the decimal point to the right of the msd and we multiply the number by 10 raised to the negative power that is equal to the number of positions the decimal point was moved to the right.

Example 1.39

Write the following numbers in scientific notation.

a. 0.37 b. 0.045 c. -0.00123

Solution:

For (a), the msd is 3; therefore, in accordance with Rule 19, we move the decimal point to its right one position and we multiply by 10 with exponent –1. For (b), the msd is 4 and thus we need to move two positions to the right; therefore, we multiply by 10 with exponent –2. We follow the same procedure for (c). Then,

a. $0.37 = 3.7 \times 10^{-1}$ b. $0.045 = 4.5 \times 10^{-2}$ c. $-0.00123 = -1.23 \times 10^{-3}$

1.10 Operations with Numbers in Scientific Notation

Multiplication and division* of very large or very small numbers is simplified with application of the following rule:

Rule 20: To multiply two numbers, we perform the following steps:

1. Write the numbers in scientific notation
2. Multiply the parts of the numbers without the 10s and their exponents
3. Multiply the product of Step 2 above by 10 with the exponent obtained by the sum of the exponents of the numbers

Example 1.40

Multiply $4,000,000$ by $23,000,000,000$

Solution:

We first write the given numbers in scientific notation and in accordance with Rule 20, we first multiply 4 by 2.3 which results in the product 9.2. Then, we multiply this product by 10 raised to power 16 which is sum of the exponents of the multiplier and multiplicand as shown below.

$$
\begin{array}{rcr}
4{,}000{,}000 & \longrightarrow & 4 \times 10^{6} \\
23{,}000{,}000{,}000 & \longrightarrow & \underline{2.3 \times 10^{10}} \times \\
\textbf{Product} & \longrightarrow & 9.2 \times 10^{16}
\end{array}
$$

Example 1.41

Multiply 0.00029 *by* 0.000003

Solution:

We first write the given numbers in scientific notation and, in accordance with Rule 20, we first multiply 2.9 by 3 which results in the product 8.7. Then, we multiply this product by 10 raised to power −10 which is the sum of the exponents of the multiplier and multiplicand as shown below.

* *Addition and subtraction of numbers in scientific notation is impractical. Accordingly, no examples are given so that they will not cause confusion with multiplication and division. Nonetheless, the procedure is as follows:*

*1. Write the numbers to be added or subtracted in the **same** power of 10.*

2. For addition, add the numbers without the 10s and their exponents. For subtraction, perform subtraction without the 10s and their exponents.

*3.Multiply the result (sum or difference) by 10 raised to the **same** power of step 1.*

$$\begin{array}{rcr} 0.00029 & \longrightarrow & 2.9 \times 10^{-4} \quad \times \\ 0.000003 & \longrightarrow & \underline{3 \times 10^{-6}} \\ \textbf{Product} & \longrightarrow & 8.7 \times 10^{-10} \end{array}$$

Four more examples follow.

a. Multiplication of *0.000025* by *8, 000, 000*

$$\begin{array}{rcr} 0.000025 & \longrightarrow & 2.5 \times 10^{-5} \quad \times \\ 8{,}000{,}000 & \longrightarrow & \underline{8 \times 10^{6}} \\ \textbf{Product} & \longrightarrow & 20 \times 10 = 2 \times 10^{2} \end{array}$$

b. Multiplication of *80, 000* by *–0.0875*

$$\begin{array}{rcr} 80{,}000 & \longrightarrow & 8 \times 10^{4} \quad \times \\ -0.0875 & \longrightarrow & \underline{-8.75 \times 10^{-2}} \\ \textbf{Product} & \longrightarrow & -70 \times 10^{2} = -7 \times 10^{3} \end{array}$$

c. Multiplication of *–250, 000* by *0.025*

$$\begin{array}{rcr} -250{,}000 & \longrightarrow & -2.5 \times 10^{5} \quad \times \\ 0.025 & \longrightarrow & \underline{2.5 \times 10^{-2}} \\ \textbf{Product} & \longrightarrow & -6.25 \times 10^{2} \end{array}$$

d. Multiplication of *–0.00125* by *–0.00008*

$$\begin{array}{rcr} -0.00125 & \longrightarrow & -1.25 \times 10^{-3} \quad \times \\ -0.00008 & \longrightarrow & \underline{-8 \times 10^{-5}} \\ \textbf{Product} & \longrightarrow & 10 \times 10^{-8} = 10^{-7} \end{array}$$

Rule 21: To divide two numbers, we perform the following steps:

1. Write the numbers in scientific notation
2. Divide the parts of the numbers without the 10s and their exponents
3. Multiply the product of Step 2 above by 10 raised to the power obtained by subtracting the exponent of the divisor from the exponent of the dividend.

Example 1.42

Divide *45, 000, 000* by *1, 500, 000*

Solution:

We first write the given numbers in scientific notation; then, in accordance with Rule 21, we first divide 4.5 by 1.5 which results in the quotient 3. Next, we multiply this product by 10 raised to power 1 which is obtained by subtracting the exponent 6 of the divisor from the exponent 7 of the dividend as shown below.

$$\begin{array}{rcl} \dfrac{45{,}000{,}000}{1{,}500{,}000} & \begin{array}{c}\longrightarrow \\ \longrightarrow\end{array} & \dfrac{4.5 \times 10^7}{1.5 \times 10^6} \\ \textbf{Quotient} & \longrightarrow & 3 \times 10 \end{array}$$

Two more examples follow.

$$\begin{array}{rcl} \dfrac{-2{,}000}{500{,}000} & \begin{array}{c}\longrightarrow \\ \longrightarrow\end{array} & \dfrac{-2 \times 10^3}{5 \times 10^5} \\ \textbf{Quotient} & \longrightarrow & -4 \times 10^{-3} \end{array}$$

$$\begin{array}{rcl} \dfrac{-21{,}000{,}000}{-0.0000105} & \begin{array}{c}\longrightarrow \\ \longrightarrow\end{array} & \dfrac{-2.1 \times 10^7}{-1.05 \times 10^{-5}} \\ \textbf{Quotient} & \longrightarrow & 2 \times 10^{12} \end{array}$$

Let us check the last division with Excel. We enter *–21,000,000* in A1. We observe that Excel displays this number in scientific notation, i.e., –2.1E+07. We enter *–0.0000105* in A2 and we observe that this number is also displayed in scientific notation as –1.05E-05. Generally, if the numbers are very large or very small, Excel displays them in scientific notation. Now, the selected cell is A3 where we type the formula =A1/A2; this formula instructs Excel to divide the contents of A1, i.e. the number –21,000,000, by the contents of A2, i.e., –0.0000105 and display the quotient in A3. We observe that Excel displays 2E+12.

1.11 Square and Cubic Roots

The *square root* of a number is the number that, when it is squared, i.e., when multiplied by itself, produces the number under the square root. The square root of a non-negative number can be either positive or negative. Thus, the square root of *25* is ± 5 since both values *+5* and *–5*, when squared, will produce 25. *The square root of a negative number is an imaginary number.* We will not be concerned with the square roots of negative numbers. The square root of a number may be indicated by the square root symbol or with the fractional exponent ½ or 0.5. Thus, the square root of 25 may be shown as $\sqrt{25}$, $25^{1/2}$, or $25^{0.5}$.

Example 1.43

Evaluate the following square roots

a. $\sqrt{81}$ b. $\sqrt{225}$ c. $\sqrt{0.0625}$ d. $\sqrt{17.85}$

Solution:

a. The square root of 81 is ±9 since $9 \times 9 = 81$ and $-9 \times (-9) = 81$ also. Thus,

$$\sqrt{81} = \pm 9$$

b. The square root of 225 is ±15 since $15 \times 15 = 225$ and $-15 \times (-15) = 225$ also. Therefore,

$$\sqrt{225} = \pm 15$$

c. $0.25 \times 0.25 = 0.0625$ and $-0.25 \times (-0.25) = 0.0625$ also. Thus,

$$\sqrt{0.0625} = \pm 0.25$$

d. The answer is not obvious. However, since $\sqrt{25} = \pm 5$ and $\sqrt{16} = \pm 4$, we expect that the square root of *17.85* will be some value between ± 4 and ± 5. There is a procedure for finding the square root of any number, but it is tedious and thus we will not discuss it here. It can be found in math tables books such as the *CRC Standard Mathematical Tables, CRC Press*. Instead, we will use Excel as follows:

On the *Standard Toolbar* click on ***f*x**. From the *Function Category* select *Math & Trig*. Scroll down the function name list to locate SQRT and click on it. Type 17.85 in the *Number* box and observe the answer displayed as 4.22493. We get the same answer by typing =SQRT(17.85). *Computers and calculators return absolute values only*. Thus,

$$\sqrt{17.85} = \pm 4.22493$$

The *cubic (or cube) root* of a number is a number that, when cubed, produces the number under the cubic root. The cubic root of a positive number is positive, whereas the cubic root of a negative number is negative. Thus, the cubic root of *125* is *5* since $5^3 = 125$, and the cubic root of –*125* is –*5* since $-5^3 = -125$. The cubic root of a number is represented by the cubic root symbol or with the fractional exponent ⅓. Thus, the cubic root of *27* may be written as $\sqrt[3]{27}$ or $27^{1/3}$.

Example 1.44

Evaluate the following cubic roots.

a. $\sqrt[3]{64}$ b. $\sqrt[3]{-343}$ c. $\sqrt[3]{0.003375}$ d. $\sqrt[3]{38.685}$

Solution:

a. The cubic root of 64 is +4 or simply 4 since $4 \times 4 \times 4 = 64$. It cannot also be –*4* since $-4 \times (-4) \times (-4) = -64$. Thus,

$$\sqrt[3]{64} = 4$$

b. The cubic root of -343 is -7 since $-7 \times (-7) \times (-7) = -343$. Thus,

$$\sqrt[3]{-343} = -7$$

c. The answer is not obvious. There is a procedure for finding the cubic root of any number, but it is very tedious and thus, we will not discuss it here. Instead, we will use Excel as follows:

On the *Standard Toolbar, we* click on $\boldsymbol{f_x}$. From the *Function Category,* we select *Math & Trig*. We scroll down the function name list to locate POWER and we click on it. We type *0.003375* in the *Number* box and $1/3$ in the *Power* box. Excel returns *0.15*. Thus,

$$\sqrt[3]{0.003375} = 0.15$$

d. We will also use Excel. On the *Standard Toolbar,* we click on $\boldsymbol{f_x}$. From the *Function Category,* we select *Math & Trig*. We scroll down the function name list to locate POWER and we click on it. We type *38.685* in the *Number* box and $1/3$ in the *Power* box. Excel returns *3.382*. Thus,

$$\sqrt[3]{38.685} = 3.382$$

This answer looks reasonably correct. Let us check it out by multiplication as follows:

$$3.382 \times 3.382 \times 3.382 = 38.683$$

1.12 Common and Natural Logarithms

The *common logarithm* or *base 10 logarithm* of a number, denoted as $\log_{10}$, is the *power* (*exponent*) to which the base *10* must be raised to produce that number. Thus, if a number is denoted as *M* and $\log_{10}M = x$, then $10^x = M$. For example, $\log_{10}100 = 2$ since $10^2 = 100$.

Another type of logarithm is the *natural* or *base e* logarithm where *e* = *271828...* and it is denoted as *ln* or log_e. It is the exponent to which the base *e* must be raised to produce that number. Thus, if a number is denoted as *N* and $\log_e N = \ln N = y$, then $e^y = N$. For example, $\ln 2 = \log_e 2 = 0.693$ since $e^{0.693} = 2$. The natural logarithm is mostly used in science and engineering, although it appears also in the computation of some financial functions, as we will see on the next chapter.

The logarithm (common or natural) of a positive number that is less than 1 is negative. The logarithm of a negative number is an imaginary number and, as mentioned before, we will not concerned with these numbers.

Note 1.12

For brevity, we will use *log* to represent the common (base 10) logarithm, and *ln* to represent the natural (base *e*) logarithm.

Example 1.45

Evaluate the following logarithms given that $ln10 = 2.3026$ and $ln100 = 4.6052$.

a. $log725$ b. $log0.125$ c. $ln64$ d. $ln0.75$

Solution:

a. The answer must be a number between *2* and *3* because *725* lies between *100* and *1000*, and by definition, $log100 = 2$ whereas $log1000 = 3$. We will use Excel to find the log of *725*.

On the *Standard Toolbar,* we click on f_x. From the *Function Category,* we select *Math & Trig*. We scroll down the function name list to locate LOG10 and we click on it. We type 725 in the *Number* box and Excel returns *2.86034*. Thus,

$$log725 = 2.86034$$

b. The answer must be a negative number because we are seeking for the *log* of a number that is less than *1*. We will use Excel to find the log of *0.125*.

On the *Standard Toolbar, we* click on f_x. From the *Function Category*, we select *Math & Trig*. We scroll down the function name list to locate LOG10 and click on it. We type *0.125* in the *Number* box and Excel returns *–0.90309*. Thus,

$$log0.125 = -0.90309$$

c. We make use of the fact that $ln10 = 2.3026$ and $ln100 = 4.6052$. Therefore, we expect $ln64$ to be a number greater than *2.3* and less than *4.6*. Again, we will let Excel compute the answer for us.

On the *Standard Toolbar,* we click on f_x. From the *Function Category,* we select *Math & Trig*. We scroll down the function name list to locate LN, and we click on it. We type 64 in the *Number* box, and we observe the answer to be *4.15888*. Thus,

$$ln64 = 4.15888$$

d. The answer must be a negative number because we are seeking for the *ln* of a number less than *1*. Let us again, use Excel to find the *ln0.75*.

On the *Standard Toolbar,* we click on f_x. From the *Function Category,* we select *Math & Trig*. We scroll down the function name list to locate LN, and we click on it. We type *0.75* in the *Number* box and observe the answer *–0.28768.* Thus,

$$ln0.75 = -0.28768$$

1.13 Decibel

A *decibel (dB)* is a unit that is used to express the relative difference in power or intensity, usually between two acoustic or electric signals. It is equal to ten times the common logarithm of the ratio of the two levels, that is,

$$dB = 10log\left|\frac{Level\ 1}{Level\ 2}\right| \tag{1.4}$$

Example 1.46

Compute the number of decibels (dB) if 90 represents Level 1 and 20 represents Level 2.

Solution:

$$dB = 10log\left|\frac{90}{20}\right| = 10log\left|\frac{9}{2}\right| = 10log4.5$$

and using Excel, we type =LOG(9/2) and Excel returns 0.6532. Then,

$$10log4.5 = 10 \times 0.6532 = 6.532\ dB$$

1.14 Percentages

Often, we hear that public securities, such as stocks and bonds, yield interest at a specified percentage, or banks paying or demanding interest at a specified percentage. *Percent*, denoted as %, is a number out of each hundred. For instance, the number 0.75 or $\frac{75}{100}$ is expressed as 75% since it means 75 out of 100.

Most often, we are interested in changes of percentages. For instance, to find the percent change between an old and a new value, we use the formula

$$\%\ Change = \frac{(New\ Value - Old\ Value)}{Old\ Value} \times 100^{*} \tag{1.5}$$

Example 1.47

Joe Smith bought common stock of ABC Corporation at *$15.00* per share. Three months later he sold his stock at *$21.50* per share. What was his profit expressed in percent?

Solution:

Here, the old (initial) value is $15.00 and the new is $21.50. Then, using (1.5) we get

* *The parentheses are used as a reminder that the subtraction must be performed before the multiplication by 100.*

$$\% \; Change = \frac{21.50 - 15.00}{15.00} \times 100 = \frac{6.50}{15.00} \times 100 = \frac{650}{15.00} = 43.33\%$$

In this example, the percent change turned out to be positive. Other times, it can be negative as illustrated by the following example.

Example 1.48

Jim Brown bought common stock at *$32.00* per share. Four months later he sold it at *$19.00* per share. What was his loss expressed in percent?

Solution:

Here, the old (initial) value is *32.00* and the new is *19.00*. Then, using (1.5) we get

$$\% \; Change = \frac{19.00 - 32.00}{32.00} \times 100 = \frac{-13.00}{32.00} \times 100 = \frac{-1300}{32.00} = -40.625\%$$

Other times, we want to find the percent error between exact (actual) and measured (experimental) values. In such as case, we express (1.5) as

$$\% \; Error = \frac{Measured \; Value - Exact \; Value}{Exact \; Value} \times 100 \tag{1.6}$$

Example 1.49

In Example 1.25, we found that the exact value of the sum of the given numbers is $\frac{72797}{18480}$ and the value returned by Excel is $\frac{3881}{938}$. Compute the percent error.

Solution:

Let us use Excel to simplify these numbers.

We start with a blank worksheet, and in A1 enter =72797/18480. Excel displays *3.939232*. In A2, we enter =3881/938 and we observe that Excel displays 4.137527. In A3, we enter the formula =(A2-A1)*100/A1. This is the formula of (1.6) above written in Excel language. Excel now displays *5.033851* and this is the percent error. Thus,

$$\% \; Error = \frac{(4.137527 - 3.939232)}{3.939232} \times 100 = 5.03\%$$

1.15 International System of Units (SI)

The *International System of Units,* denoted as *SI* in all languages, was adopted by the General Conference on Weights and Measures in 1960. It is used extensively by the international scientific community. It was formerly known as the *Metric System.* The SI system basic units are listed in Table 1.1.

The basic units of the SI system are often expressed in smaller or larger units by various powers of 10 known as *standard prefixes*. The *common prefixes* are listed in Table 1.2, and the less frequently used are listed in Table 1.3.

Table 1.4 shows some conversion factors between the SI and the English system. For other conversions, the Microsoft Excel CONVERT function can be used. This will be discussed next.

Table 1.5 shows typical temperature values in degrees Fahrenheit and the equivalent temperature values in degrees Celsius and degrees Kelvin.

Other units used in physical sciences and electronics are derived from the SI base units and the most common are listed in Table 1.6.

TABLE 1.1 *SI Base Units*

Unit of	***Name***	***Abbreviation***
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Degrees Kelvin	°K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd
Plane Angle	Radian	rad
Solid Angle	Steradian	sr

TABLE 1.2 *Most Commonly Used SI Prefixes*

Value	***Prefix***	***Symbol***	***Example***
10^9	Giga	G	12 GHz (Gigahertz) = 12×10^9 Hz
10^6	Mega	M	25 MΩ (Megaohms) = 25×10^6 Ω (ohms)
10^3	Kilo	K	13.2 KV (Kilovolts) = 13.2×10^3 volts
10^{-2}	centi	c	2.8 cm (centimeters) = 2.8 x 10^{-2} meter
10^{-3}	milli	m	4 mH (millihenries) = 4×10^{-3} henry
10^{-6}	micro	μ	6 μw (microwatts) = 6×10^{-6} watt
10^{-9}	nano	n	2 ns (nanoseconds) = 2×10^{-9} second
10^{-12}	pico	p	3 pF (picofarads) = 3×10^{-12} Farad

TABLE 1.3 *Less Frequently Used SI Prefixes*

Value	***Prefix***	***Symbol***	***Example***
10^{18}	Exa	E	1 Em (Exameter) = 10^{18} meters
10^{15}	Peta	P	5 Pyrs (Petayears) = 5×10^{15} years
10^{12}	Tera	T	3 T$ (Teradollars) = 3×10^{12} dollars
10^{-15}	femto	f	7 fA (femtoamperes) = 7 x 10^{-15} ampere
10^{-18}	atto	a	9 aC (attocoulombs) = 9×10^{-18} coulomb

TABLE 1.4 *Conversion Factors*

1 in. (inch)	2.54 cm (centimeters)
1 mi. (mile)	1.609 Km (Kilometers)
1 lb. (pound)	0.4536 Kg (Kilograms)
1 qt. (quart)	946 cm3 (cubic centimeters)
1 cm (centimeter)	0.3937 in. (inch)
1 Km (Kilometer)	0.6214 mi. (mile)
1 Kg (Kilogram)	2.2046 lbs (pounds)
1lt. (liter) = 1000 cm^3	1.057 quarts
1 Å (Angstrom)	10^{-10} meter
1 μm (micron)	10^{-6} meter

TABLE 1.5 *Temperature Scales Equivalents*

°F	°C	°K
–523.4	–273	0
32	0	273
0	–17.8	255.2
77	25	298
98.6	37	310
212	100	373

TABLE 1.6 *SI Derived Units*

Unit of	***Name***	***Formula***
Force	Newton (N)	**N = Kg·m ⁄ s^2**
Pressure	Pascal (Pa)	**Pa = N ⁄ m^2**
Stress	Pascal (Pa)	**Pa = N ⁄ m^2**
Work	Joule (J)	**J = N·m**
Energy	Joule (J)	**J = N·m**
Power	Watt (W)	**W = J ⁄ s**
Voltage	Volt (V)	**V = W ⁄ A**
Resistance	Ohm (Ω)	**Ω = V ⁄ A**
Conductance	Siemens (S)	**S = A ⁄ V**
Capacitance	Farad (F)	**F = A·s ⁄ V**
Inductance	Henry (H)	**H = V·s ⁄ A**
Frequency	Hertz (Hz)	**Hz = 1 ⁄ s**
Quantity of Electricity	Coulomb (C)	**C = A·s**
Magnetic Flux	Weber (Wb)	**Wb = V·s**
Magnetic Flux Density	Tesla (T)	**T = Wb ⁄ m^2**
Luminous Flux	Lumen (lm)	**lm = cd·sr**
Illuminance	Lux (lx)	**lx = lm ⁄ m^2**
Radioactivity	Becquerel (Bq)	**Bq = s^{-1}**
Radiation Dose	Gray (Gy)	**Gy = J ⁄ Kg**
Volume	Litre (L)	**L = m^3 × 10^{-3}**

We will conclude this chapter with a few examples, to illustrate the use of the CONVERT function in Excel.

Example 1.50

Use the Excel CONVERT function to convert *54* °*F* to degrees Celsius.

Solution:

On the *Standard Toolbar*, we click on f_x. From the *Function Category, we* select *Engineering*. We scroll down the function name list to locate CONVERT, and we click on it. We type *54* in the *Number* box, *F* in the *From_Unit* box and C in the *To_Unit* box. We click on OK, and we observe the answer *12.22* °C rounded to two decimal places.

Example 1.51

Use the Microsoft Excel CONVERT function to convert *42* °C to degrees Fahrenheit.

Solution:

On the *Standard Toolbar,* we click on f_x. From the *Function Category, we* select *Engineering*. We scroll down the function name list to locate CONVERT, and we click on it. We type *42* in the Number box, C in the *From_Unit* box and *F* in the *To_Unit* box. We click on OK, and we observe the answer *107.6 °F.*

Note 1.13

To find out what entries are allowed on the *From_Unit* and *To_unit* boxes, when using the CONVERT function, we click on *Help* on the *Standard Toolbar,* and on the pull-down menu, we click on *Contents and Index.* We select *Index* on the upper left corner, and we scroll down to CONVERT *Worksheet Function.* We click on *Display,* and we see a detailed description of the CONVERT function. We also see a list of text that CONVERT accepts as valid entries for the *From_Unit* and *To_unit* boxes.

Example 1.52

Use the CONVERT function to convert *55* miles to kilometers.

Solution:

On the *Standard Toolbar* we click on f_x. From the *Function Category,* we select *Engineering*. We scroll down the function name list to locate CONVERT, and we click on it. We type *55* in the *Number* box, *mi* in the *From_Unit* box and *m* in the *To_Unit* box. We click on OK and we observe the answer 88514 meters or 88.514 Km.

1.16 Graphs

A *graph* shows two or more related quantities in pictorial form. Two common forms of graphs are the *Cartesian* or *Rectangular form* and the *Polar form*. Figure 1.2 shows a Cartesian form of a graph with the horizontal axis labeled *x axis (abscissa*) and the vertical axis labeled *y axis (ordinate*). The abscissa and ordinate are the *co-ordinates* of the Cartesian graph. The 0 (zero) point is the intersection of the co-ordinates.

All values to the right of the zero point, represent positive values of *x* while those to the left, represent negative values. Likewise, all values above the zero point represent positive values of *y*, while those below are negative values. Any point on the plane, formed by the *x* and *y* axes is designated as $P(x,y)$. Thus, point *A* is identified as $A(3,2)$ since it is located *3* units to the right of the origin *(zero)*, and *2* units above the origin in the *y* axis direction. Likewise, point *B* is identified as B(–2,4) since it is located *2* units to the left of the origin *(zero)* in the *x* axis, and *4* units above the origin in the *y* axis direction. Points C and *D* are identified similarly.

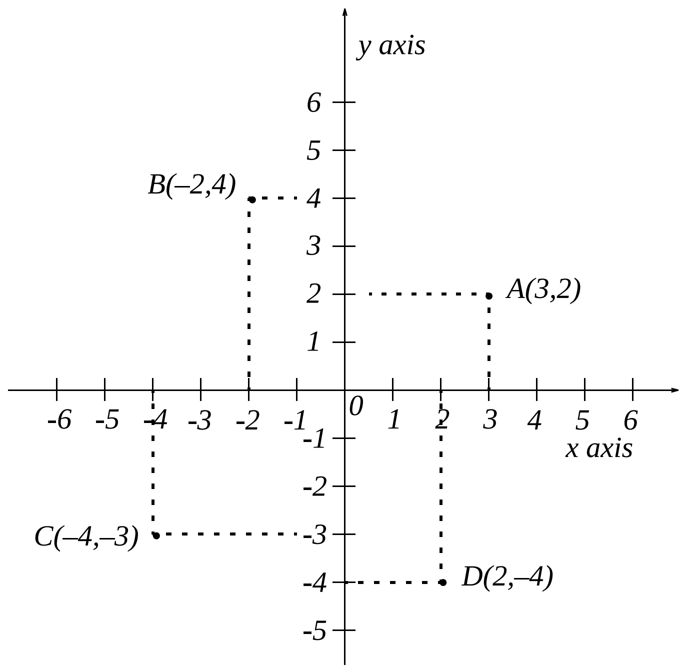


Figure 1.2. Cartesian (rectangular) co-ordinates graph

Figure 1.3 shows a *polar form* of a graph. Any point on this graph, is identified by its distance from the origin, and the angle that the line connecting the origin and that point, makes with the horizontal line of the graph, starting at zero degrees point and rotating counterclockwise. The polar form is used in navigation and radar detection, and we will not be concerned with this type.

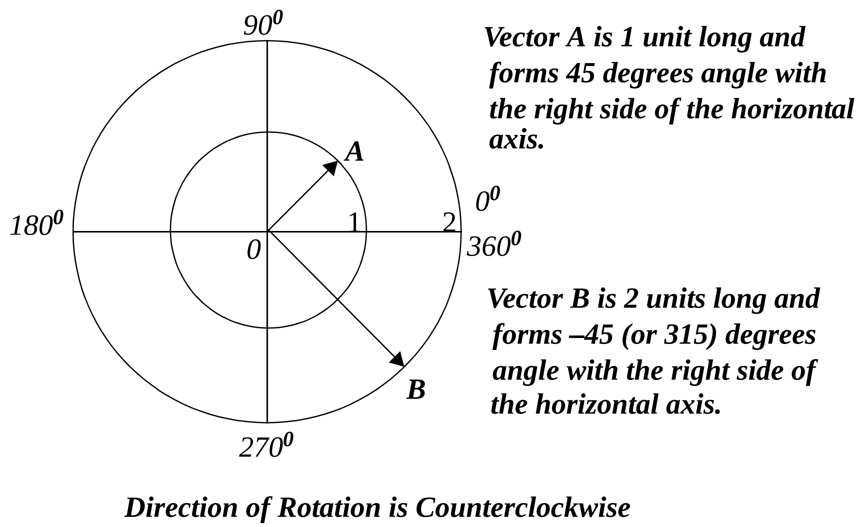


Figure 1.3. Polar graph

Any line from the origin to any other point is a vector. A *vector* is a line that is specified by its length from a reference point, normally the origin, and the angle it forms with a reference axis, normally the horizontal axis. Thus, $\mathbf{A}$ and $\mathbf{B}$ in Figure 1.3 are vectors. A *phasor* is a rotating vector. For example, if vector $\mathbf{A}$ rotates (normally counterclockwise), then it is referred to as a phasor.

Figure 1.4 shows a graph with a *constant value* of 12 units. This graph is referred to as a *uniform function,* since the number of units does not change with time. Thus, if the ordinate v represents a physical quantity such as the voltage of an automobile battery, we see that this voltage has the same value of *12* volts for all times.

Figure 1.5 shows a *linear graph*. It is called linear because there is a linear (straight line) relationship between the quantities represented by the coordinates. Linear graphs have the property that equal increments in the abscissa produce equal increments on the ordinate axis.

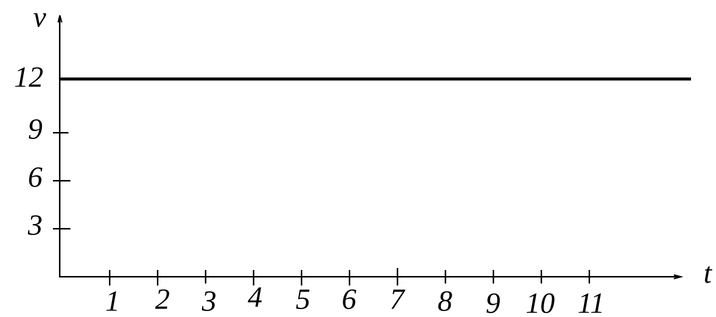


Figure 1.4. *Graph of a uniform function*

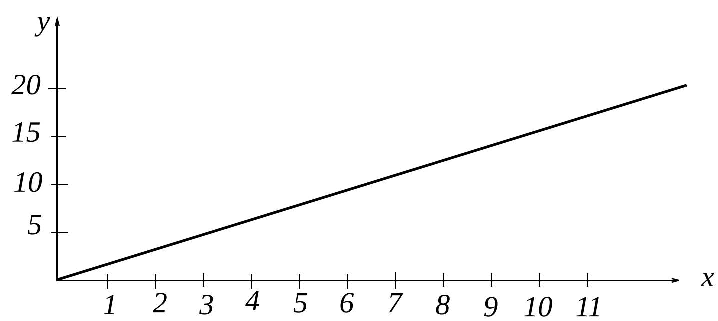


Figure 1.5. *Linear graph*

Figure 1.6 shows two *non-linear graphs* on the same set of a Cartesian form. As indicated, one is a *rising exponential,* and the other a *decaying exponential*. We will encounter a rising exponential curve in Chapter 9 when we discuss the exponential distribution. Decaying exponential curves are used in many applications such as in *radioactivity,* which is a topic discussed in physics and chemistry. For instance, the decay rate of radioactive elements can be represented by decaying exponentials. The isotope* *Thorium–234*, for example, decays to half of its original radioactivity in *25* days, and thus, it is said to have a half-life of 25 days. However, *Thorium–232* has a half-life of approximately 14 billion years.

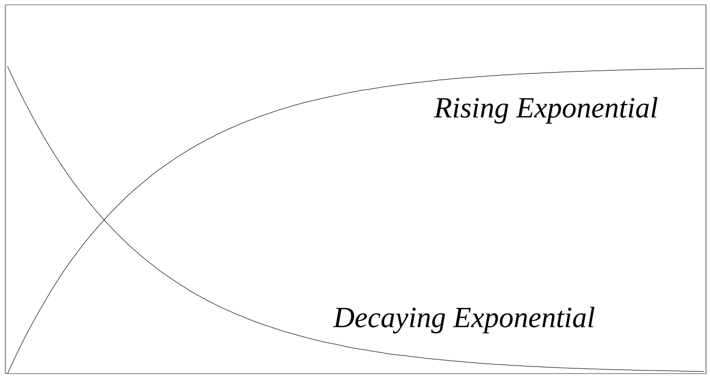


Figure 1.6. Non-linear (exponential) graphs

Non-linear graphs are those that do not produce equal increments on the ordinate axis for equal increments on the abscissa.

Figure 1.7 is a *sinewave graph,* which is also a non-linear graph. Sinewaves are used extensively in AC Electronics to represent time-varying quantities such as voltage and current.

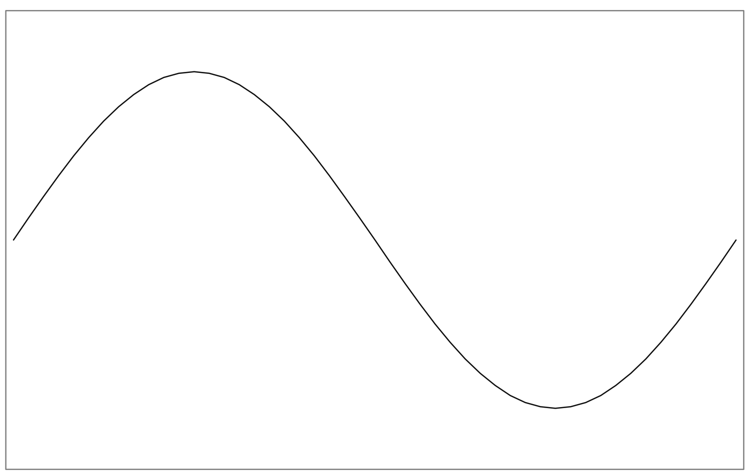


Figure 1.7. Sinewave graph.

* *Isotopes are discussed in physics and chemistry textbooks. Briefly, an isotope is one of two or more atoms having the same atomic number but different mass numbers.*

1.17 Summary

- The *decimal* (*base 10*) number system uses the *digits* (numbers) *0, 1, 2, 3, 4, 5, 6, 7, 8*, and *9*. This is the number system we use in our everyday arithmetic calculations such as the monetary transactions.
- A *positive number* is a number greater than zero and it is understood to have a plus (+) sign in front of it. The (+) sign in front of a positive number is generally omitted. Thus, any number without a sign in front of it is understood to be a positive number.
- A *negative number* is less than zero and it is written with a minus (–) sign in front of it. The minus (–) sign in front of a negative number is a must; it is necessary so that we can distinguish negative numbers from the positive numbers.
- Positive and negative numbers can be whole (integer) or fractional numbers. To avoid confusion between the addition operation (+) and positive numbers, which are also denoted with the (+) sign, we enclose positive numbers with their sign inside parentheses whenever necessary. Likewise, we enclose negative numbers in parentheses to distinguish them from the subtraction (–) symbol.
- The number *0 (zero)* is considered neither positive nor negative; it is the number that separates the negative from the positive numbers.
- The positive and negative numbers we are familiar with, are referred to as the *real numbers.*
- The *absolute value* of a number is that number without a positive or negative sign, and is enclosed in small vertical lines. Thus, the absolute value of X is written as $|X|$.
- To add numbers with the same sign, we add the absolute values of these numbers and place the common sign (+ or –) in front of the result (sum). We can omit the plus sign in the result if positive. We must not omit the minus sign if the result is negative.
- For the subtraction $A–B$, the number A is called the *minuend* and the number B is called the *subtrahend*. The result of the subtraction is called the *difference*.
- To add two numbers with different signs, we subtract the number with the smaller absolute value from the number with the larger absolute value, and we place the sign of the larger number in front of the result (sum).
- To subtract one number (the subtrahend) from another number (the minuend) we change the sign (we replace + with – or – with +) of the subtrahend, and we perform addition instead of subtraction.
- Let X be any number; then, the following relations apply for the possible combinations of addition and subtraction of positive and negative numbers.

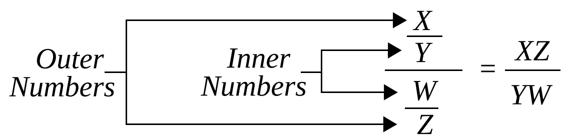
$$+(+X) = +X = X$$
$$+(-X) = -X$$
$$-(+X) = -X$$
$$-(-X) = X$$

- When two numbers with the same sign are multiplied, the product will be positive $(+)$. If the numbers have different signs, the product will be negative $(-)$.
- A *rational number* is a number that can be expressed as an integer or a quotient of integers, excluding zero as a denominator. In division, the number that is to be divided by another is called the *dividend*. For the ratio form A/B the dividend is A. The dividend is also referred to as the *numerator*. The number by which the dividend is to be divided, is called the *divisor*. For the ratio form A/B the divisor is B. The divisor is also called the *denominator*. The number obtained by dividing one quantity by another is the *quotient*.
- When two numbers with the same sign are divided, the quotient will be positive $(+)$. If the numbers have different signs, the quotient will be negative $(-)$.
- A *decimal point* is the dot in a number that is written in decimal form. Its purpose is to indicate where values change from positive to negative powers of 10. Every number has a decimal point although it may not be shown.
- A number is classified as an *integer*, *fractional* or *mixed* depending on the position of the decimal point in that number.
- An *integer* number is a whole number and it is understood that its decimal point is positioned after the last (rightmost) digit although not shown.
- A *fractional* number, positive or negative, is always a number less than one. It can be expressed in a rational form such as $3/4$ or in *decimal point form* as 0.75. When a fractional number is written in decimal point form, the decimal point appears in front of the first (leftmost) digit.
- A *mixed number* consists of an integer and a fractional number separated by the decimal point. For instance, the number 409.0875 consists of the integer part 409 and the fractional part 0.0875.
- Any number (integer, fractional or mixed) may be written in a rational form where the numerator (dividend) is the number itself and the denominator (divisor) is 1.
- The *reciprocal* of an integer number is a fraction with numerator 1 and denominator that number itself. Since any number can be considered as a fraction with denominator 1, the reciprocal of a fraction is another fraction with the numerator and denominator reversed. The product (multiplication) of any number by its reciprocal is 1.

- The value of a rational number is not changed if both numerator and denominator are multiplied by the same number.
- To add two or more fractional numbers in rational form, we first express the numbers in a common denominator; then, we add the numerators to obtain the numerator of the sum. The denominator of the sum is the common denominator. If the numbers are in decimal point form, we first align the given numbers with the decimal point, and we add the numbers as it is done in normal addition.
- The *Least Common Multiple (LCM)* is the smallest quantity that is divisible by two or more given quantities without a remainder.
- To subtract a number given in rational form from another number of the same form, we first express the numbers in a common denominator; then, we subtract the numerator of the subtrahend from the numerator of the minuend to obtain the numerator of the difference. The denominator of the difference is the common denominator. If the numbers are in decimal point form, we first align the given numbers with the decimal point; then, we subtract the subtrahend from the minuend as it is done in normal subtraction.
- To multiply a number given in a rational form by another number of the same form, we multiply the numerators to obtain the numerator of the product. Then, we multiply the denominators to obtain the denominator of the product. If the numbers are in decimal point form, we multiply the given numbers as is done in normal multiplication, and we place the decimal point at the position that is equal to the sum of the decimal places of the given numbers.
- To divide a number (dividend) given in a rational form, by another number (divisor) of the same form, we multiply the numerator of the dividend by the denominator of the divisor to obtain the numerator of the quotient. Then, we multiply the denominator of the dividend by the numerator of the divisor to obtain the denominator of the quotient. If the numbers are in decimal point form, we first express them in rational form and convert to integers by multiplying both numerator and denominator by the same smallest possible number that will make them integer numbers; then, we perform the division.
- A *complex fraction* is a fraction whose numerator or denominator or both are fractions. The general form of a complex fraction is

$$\frac{\frac{X}{Y}}{\frac{W}{Z}}$$

Any complex fraction can be replaced by a simple fraction whose numerator is the product of the outer numbers X and Z, and whose denominator is the product of the inner numbers Y and W, as shown in the sketch below.



- In the mathematical expression $\boldsymbol{a^n}$, $\boldsymbol{a}$ is the *base*, $\boldsymbol{n}$ is the *exponent* or *power,* and $\boldsymbol{a^n}$ is said to be a *number in exponential form.* The exponent indicates the number of times the base appears in a string of multiplication operations.
- Any number written without an exponent is understood to have exponent 1.
- Any number that has exponent 0 is equal to 1.
- Any number may be written in powers of 10 multiplied by the appropriate number of digits.
- Any number can be replaced by its reciprocal if the sign of its exponent is changed.
- In any number, the digit immediately to the left of the decimal point occupies the *units* position, the second digit occupies the *tens* position, the third the *hundreds* position, the fourth the *thousands* position and so on. The leftmost digit is the *most significant digit (msd)* and the rightmost is the *least significant digit (lsd)*.
- A number is said to be written in scientific notation when the decimal point appears immediately to the right of the msd.
- To write an integer or a mixed number in scientific notation, we move the decimal point to the left until we reach the msd. We place it immediately to the right of the msd. Then, we multiply the new number by 10 raised to the power that is equal to the number of positions the decimal point was moved to the left.
- To write a fractional number in scientific notation, we move the decimal point to the right of the msd and we multiply the number by 10 raised to the negative power that is equal to the number of positions the decimal point was moved to the right.
- Multiplication of very large or very small numbers is simplified with application of the following rule:

  1. Write the numbers in scientific notation
  2. Multiply the parts of the numbers without the 10s and their exponents
  3. Multiply the product of Step 2 above by 10 with the exponent obtained by the sum of the exponents of the numbers

- Division of very large or very small numbers is simplified with application of the following rule:

  1. Write the numbers in scientific notation
  2. Divide the parts of the numbers without the 10s and their exponents

3. Multiply the product of Step 2 above by 10 raised to the power obtained by subtracting the exponent of the divisor from the exponent of the dividend.

- The *square root* of a number is the number that, when it is squared, i.e., when multiplied by itself, produces the number under the square root. The square root of a non-negative number can be either positive or negative. *The square root of a negative number is an imaginary number.*
- The *cubic (or cube) root* of a number is a number that, when cubed, produces the number under the cubic root. The cubic root of a positive number is positive, whereas the cubic root of a negative number is negative. The cubic root of a number A is represented by the cubic root symbol $\sqrt[3]{A}$ or with the fractional exponent ⅓, i.e., $A^{1/3}$.
- The *common logarithm* or *base 10 logarithm* of a number, denoted as log_{10}, is the *power* (*exponent*) to which the base *10* must be raised to produce that number. Thus, if a number is denoted as M and $log_{10}M = x$, then $10^x = M$.
- The *natural* or *base e* logarithm where $e = 271828...$ denoted as *ln* or log_e, is the exponent to which the base *e* must be raised to produce that number. Thus, if a number is denoted as *N* and $log_eN = lnN = y$, then $e^y = N$.
- The logarithm (common or natural) of a positive number that is less than 1 is negative. The logarithm of a negative number is an imaginary number.
- A *decibel (dB)* is a unit that is used to express the relative difference in power or intensity, usually between two acoustic or electric signals. It is equal to ten times the common logarithm of the ratio of the two levels, that is,

$$dB = 10log\left|\frac{Level\ 1}{Level\ 2}\right|$$

- *Percent*, denoted as %, is a number out of each hundred. The percent change between an old and a new value is found by application of the formula

$$\%\ Change = \frac{(New\ Value - Old\ Value)}{Old\ Value} \times 100$$

- The percent error between exact (actual) and measured (experimental) values can be found by application of the formula

$$\%\ Error = \frac{Measured\ Value - Exact\ Value}{Exact\ Value} \times 100$$

- The *International System of Units,* denoted as *SI* in all languages, was adopted by the General Conference on Weights and Measures in 1960. It is used extensively by the international scientific community. It was formerly known as the *Metric System.*

1.18 Exercises

1. Write the following numbers in scientific notation.

   a. $54,000$ b. $67,000,000$ c. $89,000,000,000$

   d. 0.0045 e. 0.0000076 f. 0.00000000098

2. Perform the following multiplications. The answers need not be in scientific notation.

   a. $(1.2 \times 10^{5}) \times (2.1 \times 10^{8})$ b. $(3.4 \times 10^{7}) \times (4.3 \times 10^{-2})$

   c. $(5.6 \times 10^{-9}) \times (6.5 \times 10^{6})$ d. $(7.9 \times 10^{-12}) \times (9.8 \times 10^{-6})$

3. Perform the following divisions. The answers need not be in scientific notation.

   a. $(5 \times 10^{8}) \div (1.25 \times 10^{4})$ b. $(7.5 \times 10^{3}) \div (2.5 \times 10^{9})$

   c. $(8 \times 10^{-7}) \div (2 \times 10^{5})$ d. $(9 \times 10^{-6}) \div (3 \times 10^{-9})$

4. Perform the following temperature scale conversions. Round your answers to the nearest integer.

   a. 40 °C to °F b. 60 °F to °C c. 50 °C to °K

   d. 320 °K to °C e. 90 °F to °K f. 350 °K to °F

1.19 Solutions to Exercises

Dear Reader:

The remaining pages on this chapter contain the solutions to the exercises.

You must, for your benefit, make an honest effort to find the solutions to the exercises without first looking at the solutions that follow. It is recommended that first you go through and work out those you feel that you know. For the exercises that you are uncertain, review this chapter and try again. Refer to the solutions as a last resort and rework those exercises at a later date.

You should follow this practice with the rest of the exercises of this book.

1.

a. $54,000 = 5.4 \times 10^4$ b. $67,000,000 = 6.7 \times 10^7$ c. $89,000,000,000 = 8.9 \times 10^{10}$

d. $0.0045 = 4.5 \times 10^{-3}$ e. $0.0000076 = 7.6 \times 10^{-6}$ f. $0.00000000098 = 9.8 \times 10^{-10}$

2.

a. $1.2 \times 2.1 = 2.52$, $5 + 8 = 13$, and thus $(1.2 \times 10^5) \times (2.1 \times 10^8) = 2.52 \times 10^{13}$

b. $3.4 \times 4.3 = 14.62$, $7 + (-2) = 5$, and thus $(3.4 \times 10^7) \times (4.3 \times 10^{-2}) = 14.62 \times 10^5$

c. $5.6 \times 6.5 = 36.4$, $-9 + 6 = 3$, and thus $(5.6 \times 10^{-9}) \times (6.5 \times 10^6) = 36.4 \times 10^3$

d. $7.9 \times 9.8 = 77.42$, $-12 + (-6) = -18$, and thus $(7.9 \times 10^{-12}) \times (9.8 \times 10^{-6}) = 77.42 \times 10^{-18}$

3.

a. $5/1.25 = 4$, $8 - 4 = 4$, and thus $(5 \times 10^8) \div (1.25 \times 10^4) = 4 \times 10^4$

b. $7.5/2.5 = 3$, $3 - 9 = -6$, and thus $(7.5 \times 10^3) \div (2.5 \times 10^9) = 3 \times 10^{-6}$

c. $8/2 = 4$, $-7 - 5 = -12$, and thus $(8 \times 10^{-7}) \div (2 \times 10^5) = 4 \times 10^{-12}$

d. $9/3 = 3$, $-6 - (-9) = 3$, and thus $(9 \times 10^{-6}) \div (3 \times 10^{-9}) = 3 \times 10^3$

4. Using the procedures that we used in Examples 1.50 and 1.51 we get:

a. =CONVERT(40,"C","F")=104 °F

b. =CONVERT(60,"F","C")= 16 °C

c. =CONVERT(50,"C","K")= 323 °K

d. =CONVERT(320,"K","C")= 47 °C

e. =CONVERT(90,"F","K")=305 °K

f. =CONVERT(350,"K","F")=170 °F