1.3 Determinants of Matrices of Higher Order

Exercises

1. Compute the cofactors of the following matrices

   (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$;

   (b) $B = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -4 & 2 \\ -1 & 2 & -6 \end{bmatrix}$;

   (c) $C = \begin{bmatrix} 2 & -1 & 0 & -3 \\ -1 & 1 & 0 & -1 \\ 4 & 0 & 3 & -2 \\ -3 & 0 & 0 & 1 \end{bmatrix}$;

   (d) $D = \begin{bmatrix} 3 & 5 & 1 & 0 \\ 2 & 1 & 4 & 5 \\ 1 & 7 & 4 & 2 \\ -3 & 5 & 1 & 1 \end{bmatrix}$.

2. Compute the determinants of matrices A, B, C, D from exercise 1.

3. Solve the following system of four linear equations

$$\begin{aligned} -x_1 + 2x_2 - 3x_3 + 5x_4 &= 14 \\ x_1 + 3x_2 + 2x_3 - x_4 &= 9 \\ 3x_1 - 3x_2 + 2x_3 + 4x_4 &= 19 \\ 4x_1 + 2x_2 + 5x_3 + x_4 &= 27. \end{aligned}$$

4. In an automobile dealership, the most popular passenger cars are Brand A, B, C and D. Because buyers bargain for the best price, the sales price for each brand is not the same. The following table shows the sales and revenues for a 4-month period. Compute the average sales price for each of these brands of cars.

Month	Brand A	Brand B	Brand C	Brand D	Revenue
1	25	60	50	10	$3,235,000$ €
2	28	42	58	13	$2,870,000$ €
3	45	53	56	15	$3,420,000$ €
4	45	50	50	15	$3,235,000$ €

5. A corporation is made of four departments. Suppose that some of the goods being produced are consumed in the production process. Productivity of each department influences the requirements from other departments based on the following table, showing the amount of good of Department i used to produce one unit of good of Department j, and the amount of final product for each department. Compute the production

Dept.	Coefficients				Final Product
	1	2	3	4	
1	0.6	0.002	0	0	1000
2	0.1	0.2	0	0.005	1000
3	0	0.1	0.3	0	1000
4	0.2	0.3	0.4	0.6	1000

   volume of each department.

6. Prove that the definition of the determinant of a square matrix of order n, given in this section, for $n = 2$ coincides with the definition of the determinant of a square matrix of order 2 given in the previous section.

7. Prove that the definition of the determinant of a square matrix of order n, given in this section, for $n = 3$ coincides with the definition of the determinant of a square matrix of order 3 given in the previous section.

8. Consider a Leontief input-output model of an economy with tow goods, steel and coal. Suppose 0.1 units of steel and 2 units of coal are required to produce 1 unit of steel, and 0.3 units of steel are required to produce 1 unit of coal. Suppose also that the external demand for the goods is 12 units of steel and 3 units of coal. Find the production schedule which meets this demand.

9. Suppose an economy has three goods – cement, electricity, and steel. The following table shows the amounts of the goods used to produce one unit of each good, and the amounts of final products.

Good	**Coefficients**			**Final Product**
	Cement	Electricity	Steel	
Cement	0.2	0.6	0.2	30
Electricity	0	0.1	0.4	20
Steel	0.6	0	0.3	40