1.4 Matrix Multiplication. The Inverse of a Matrix

Exercises

1. For matrices A, B, C, D given by

$$A = \begin{bmatrix} 1 & -4 & -1 \\ -2 & 0 & 3 \\ 7 & -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 3 & 1 \\ -2 & 1 & 1 \\ 6 & -1 & 4 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & -3 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$$

   determine if the following operations are possible, and (if they are) find the results.

   (a) AB;
   (b) AC;
   (c) BD;
   (d) CD;
   (e) BA;
   (f) CA;
   (g) DA.

2. Solve the system

$$\begin{aligned} 2x_1 + x_2 - x_3 + x_4 - 3x_5 &= 7 \\ x_1 + 2x_3 - x_4 + x_5 &= 2 \\ -2x_2 - x_3 + x_4 - x_5 &= -5 \\ 3x_1 + x_2 - 4x_3 + 5x_5 &= 6 \\ x_1 - x_2 - x_3 - x_4 + x_5 &= 3 \end{aligned}$$

3. Solve the system

$$\begin{aligned} 4x_1 - x_2 + x_3 &= 8 \\ 2x_1 + 5x_2 + 2x_3 &= 3 \\ x_1 + 2x_2 + 4x_3 &= 11 \end{aligned}$$

   (a) by Gauss' Elimination;

   (b) by Cramer's Rule;

   (c) by the inverse matrix method.

4. A small businessman allocates his time among sales (both new clients and old clients), office management, and long-range planning. He decides that he should devote half his time to sales and twice as much time to old clients as to new clients. Also, he decides to devote twice as much time to new clients as to long planning. Assume he works 40 hours each week. How should the businessman allocate his time to meet his goals?

5. The Inshur & Bshur Insurance Agency records the sales made by each agent on a large bulletin board. The agency has a sales staff of five, and it offers life insurance, automobile insurance, and home insurance. The bulletin board can be organized as shown in the following table. The

Type of Policy	**Alice**	**Barbara**	**Charles**	**David**	**Ellen**
Life	8	10	6	3	5
Automobile	3	4	12	12	10
Home	7	6	3	2	8

   data in the table are for October. If we agree to remember the meaning of the entry in each location, then the sales figures for March can be represented by a 3×5 matrix

$$A = \begin{bmatrix} 8 & 10 & 6 & 3 & 5 \\ 3 & 4 & 12 & 12 & 10 \\ 7 & 6 & 3 & 2 & 8 \end{bmatrix}.$$

The corresponding matrix for April is

$$B = \begin{bmatrix} 4 & 6 & 5 & 6 & 4 \\ 5 & 2 & 2 & 6 & 8 \\ 6 & 8 & 5 & 2 & 6 \end{bmatrix}.$$

(a) Find the matrix representing the combined sales March and April.

(b) If the sales manager sets a goal for May for each salesperson which is twice the April sales for each type of policy, find the matrix representing the goal.

6. In the situation described in exercise 5, a goal is set for the sales in the quarter June through September to be equal to the sum of the sales in March and twice those in April. Find the matrix representing the goal.

7. In the situation described in exercise 5, suppose that the annual prices are 1,000 € for a life policy, 600 € for an automobile policy, and 500 € for a home policy. Find the matrices C and D representing the revenue generated by each salesperson for each type of policy in March and April. Find the matrix P such that $C = PA$ and $D = PB$.

8. In the situation described in exercise 7, suppose a goal is set for the revenues in May to be 10% higher than those in April. Find the matrix representing the increased revenues.

9. In the situation described in exercise 7, suppose the sales manager sets a goal for revenues in May compared to those in April, which is 10% increase of revenue from each type of policy for Alice, and 15% increase of revenue from each type of policy for David. Find the matrix R such that the revenues matrix for May is $E = DR$.

10. Which square matrix $D = [d_{ij}]$ of order n has the following properties

$$d_{ij} = 1 \quad \text{for } i = j$$

and

$$d_{ij} = 0 \quad \text{for } i \neq j?$$

11. Show that for every $m \times n$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

holds

$$AI_n = I_m A = A,$$

where I_m and I_n are the identity matrices of orders m and n.