

2.2 Compound Interest

Exercises

1. Find the future value if 12,000 € is invested for 5 years at 8%, compounded
 - (a) annually;
 - (b) semiannually;
 - (c) quarterly;
 - (d) monthly?
2. What interest will be earned if 3,800 € is invested for 6 years at 6%, compounded
 - (a) annually;
 - (b) semiannually;
 - (c) quarterly;
 - (d) monthly?
3. What present value amounts to 6,800 € if it was invested for 4 years at 6%, compounded annually?
4. What present value amounts to 1,000 € if it was invested for 4 years at 8%, compounded semiannually?
5. An amount of 5,000 € was deposited in a bank which pays 6% interest, compounded annually. After 4 years, the investor withdrew 2,000 € and after the 3 following years he withdrew another 3,000 €. How much was in the account 9 years the investment was done?
6. To purchase a building, a costumer offers 45,000 € now, and another customer offers 20,000 € now, 20,000 € after 4 years and 20,000 € after 10 years. Which offer is more convenient for the seller if she calculates 6% interest, compounded semiannually?

7. A debtor has to pay 17,400 € after 4 years and 23,500 € after 7 years. The whole credit could be returned today. What is the amount to be paid today if 6% interest, compounded semiannually, is calculated?
8. How long (in years) would an amount have to be invested at 10%, compounded annually, to double?
9. A person deposits in a bank 2,000 € at 3% and another person deposits 2,500 € at 2%. After what time would the two own the same amount if interest is compounded annually?
10. An amount of 1,000 € is invested at 6%, compounded
 - (a) once in a four months period;
 - (b) monthly;
 - (c) daily (suppose a year has 365 days);
 - (d) continuously.Find the future value after 10 years.
11. An amount of 5,000 € is invested at 10%, compounded
 - (a) annually;
 - (b) semiannually;
 - (c) daily (suppose a year has 365 days);
 - (d) continuously.Find the balance of the account after 10 years.
12. How much money should be invested today at 7%, compounded continuously, so that it will be worth 20,000 € in 20 years?
13. What is the present value of 10,000 € over a 5-year period of time if interest is compounded continuously at an annual rate of 7%? What is the present value of 20,000 € under the same conditions?
14. A sum of money is invested at a certain fixed interest rate, compounded continuously. After 10 years the money has doubled. How will the balance at the end of 20 years compare with the initial investment?

15. In 1626, Peter Minuit traded trinkets worth \$24 to a tribe of Native Americans for land on Manhattan Island. Assume that in 1990 the same land was worth \$25.2 billion. If the sellers in this transaction had invested their \$24 at 7% annual interest compounded continuously during the entire 364 years period, who would have gotten the better of the deal? By how much?
16. When a bank offers interest at an annual rate p and compounds interest more than once a year, the total interest earned during the year is greater than $p\%$ of the balance at the beginning of that year. The actual percentage by which the balance grows during a year is called the *effective interest rate*, while the advertised rate p is called the *nominal interest rate*. In other words, the effective interest rate is the simple rate that is equivalent to the nominal compound interest rate.
 - (a) If interest is compounded m times a year, show that the effective rate is $100 \left(\left(1 + \frac{p}{100m} \right)^m - 1 \right)$.
 - (b) If the interest is compounded continuously, show that the effective rate is $100 \left(e^{\frac{p}{100}} - 1 \right)$.
17. If a bank offers interest at a nominal rate of 6%, how much greater is the effective rate if the interest is compounded continuously than if the compounding is quarterly?
18. Which investment has the greater effective rate: 8.2% compounded quarterly, or 8.1% compounded continuously?
19. Show that for a certain fixed nominal interest rate, the greatest effective interest rate is achieved when the interest is compounded continuously. In other words, among all the ways that interest can be compounded, the greatest interest is earned when it is compounded continuously.
20. How quickly will money double if it is invested at an annual interest rate of 6%, compounded continuously?
21. Money deposited in a certain bank doubles every 13 years. The bank compounds interest continuously. What annual interest rate does the bank offer?