1.5 Limits of Functions

Exercises

In Exercises 1–10 compute the indicated limit if it exists.

1. $\lim_{x \to -1}(3x^2 + 5x - 2)$;
2. $\lim_{x \to 2}(x^3 - 3x^2 + 5x - 2)$;
3. $\lim_{x \to 0}(x + 1)^2(x - 1)$;
4. $\lim_{x \to 2} \frac{x-1}{x+2}$;
5. $\lim_{x \to 1} \frac{2x+3}{x-2}$;
6. $\lim_{x \to 3} \frac{(x-2)(x-3)}{(x+2)(x-3)}$;
7. $\lim_{x \to 1} \frac{x^2-1}{x^2-3x+2}$;
8. $\lim_{x \to 1} \frac{\sqrt{x}-2}{x-4}$;
9. $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$;
10. $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$.

In Exercises 11–16 determine if the given function is continuous at the specified value of x.

11. $f(x) = 3x^2 + 5x - 2$, $x = 2$;
12. $f(x) = x^3 - 3x^2 + 5x - 2$, $x = -1$;
13. $f(x) = \frac{x-1}{x+2}$, $x = 1$;

14. $f(x) = \frac{x-1}{x+2}$, $x = -2$;
15. $f(x) = \frac{2x+3}{x-2}$, $x = 3$;
16. $f(x) = \frac{2x+3}{x-2}$, $x = 2$.

Determine which of the functions described in Exercises 17–21 are continuous.

17. The number of people in a village as a function of time.
18. The weight of a baby as a function of time during the second month of the baby's life.
19. The number of pairs of pants as a function of the number of meters of cloth from which they are made. Each pair requires 3 meters.
20. The distance traveled by a car in stop-and-go traffic as a function of time.
21. You start in Portugal and drive eastward toward Turkey. Consider the function giving the local time of day as a function of your distance from your starting point.
22. If the profit function for a product is given by

$$P(x) = 92x - x^2 - 1{,}760,$$

    find $\lim_{x \to 40} P(x)$.
23. Sales y (in thousands of euros) are related to advertising expenses x (in thousands of euros) according to

$$y(x) = \frac{200x}{x+10}.$$

    Find

    (a) $\lim_{x \to 10} y(x)$;

(b) $\lim_{x \to 0} y(x)$.

24. Suppose that the cost of removing p percent of the particulate pollution from the smokestack emissions at a steel plant is

$$C(p) = \frac{7{,}300p}{100 - p}.$$

Find

(a) the cost of removing 50% of the pollution;

(b) the cost of removing 90% of the pollution;

(c) the cost of removing 99% of the pollution;

(d) the cost of removing 100% of the pollution.

25. Suppose that the cost of obtaining water that contains p percent impurities is given by

$$C(p) = \frac{120{,}000p}{p} - 1{,}200.$$

Find

(a) $\lim_{p \to 100} C(p)$;

(b) $\lim_{p \to 0} C(p)$.

Interpret the results.

26. Suppose that the weekly sales volume (in thousands of units) for a particular product is given by

$$x = \frac{32}{(p+8)^{2/5}},$$

where p is the price in euros per unit.

(a) Is this function continuous for all values of p? For what value of p is it discontinuous?

(b) Because of the practical application, for the price p we know that $p > 0$. Is this function continuous for $p > 0$?

27. Suppose that the cost of removing p percent of the particulate pollution from the exhaust gases at a an industrial site si given by

$$C(p) = \frac{8{,}100p}{100 - p}.$$

Find any discontinuities for $C(p)$.

28. Suppose that the income tax rate $R(x)$ in a country is a function of monthly income x as follows

$$R(x) = \begin{cases} 0.10 & \text{if } 0 \leq x \leq 200 \\ 0.15 & \text{if } 200 < x \leq 1{,}500 \\ 0.20 & \text{if } 1{,}500 < x \leq 3{,}000 \\ 0.25 & \text{if } 3{,}000 < x. \end{cases}$$

Identify the discontinuities for $R(x)$.