1.7 Techniques of Differentiation

Exercises

1. Differentiate the given functions:

   (a) $y = x^{-4}$;

   (b) $y = x^{7/3}$;

   (c) $y = \frac{9}{\sqrt{t}}$;

   (d) $y = \frac{3}{2t^2}$;

   (e) $y = x^2 + 2x + 3$;

   (f) $y = 3x^5 - 4x^3 + 9x - 6$;

   (g) $f(x) = x^9 - 5x^8 + x + 12$;

   (h) $f(x) = \frac{1}{4}x^8 - \frac{1}{2}x^6 - x + 2$;

   (i) $y = \frac{1}{t} + \frac{1}{t^2} - \frac{1}{\sqrt{t}}$;

   (j) $y = \frac{3}{x} - \frac{2}{x^2} + \frac{2}{3x^3}$;

   (k) $f(x) = \sqrt{x^3} + \frac{1}{\sqrt{x^3}}$;

   (l) $f(t) = 2\sqrt{t^3} + \frac{4}{\sqrt{t}} - \sqrt{2}$;

   (m) $y = -\frac{x^2}{16} + \frac{2}{x} - x^{3/2} + \frac{1}{3x^2} + \frac{x}{3}$;

   (n) $y = x^2(x^3 - 6x + 7)$;

   (o) $y = \frac{x^5 - 4x^2}{x^3}$;

2. (a) Use the graph of the function

   $$P(q) = 6q - q^2$$

   to determine whether each of the following derivatives is positive, negative, or zero: $P'(1)$, $P'(3)$, $P'(4)$. Explain.

   (b) Find $P'(q)$ and the three derivatives in part (a).

3. The consumer demand for a certain commodity is $D(p) = -200p + 12{,}000$ units per month when the market price is p euro per unit.

(a) Express consumers' total monthly expenditure $E(p)$ for the commodity as a function of p and draw the graph. (Consumers' total monthly expenditure is the total amount of money which the consumers spend each month for the commodity.)

(b) Find the rate of change of consumers' total monthly expenditure with respect to price p.

(c) What happens to consumers' total monthly expenditure when the market price is 30 € per unit?

4. An efficency study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8:00 A.M. will have assembled

$$f(t) = -t^3 + 6t^2 + 15t$$

transistor radios t hours later.

(a) Derive a formula for the rate at which the worker will be assembling radios after t hours.

(b) At what rate will the worker be assembling radios at 9:00 A.M.?

(c) How many radios will the worker actually assemble between 9:00 A.M. and 10:00 A.M.?

5. The gross annual earnings of a certain company were ishin $A(t) = 0.1t^2 + 10t + 20$ thousand euros t years after its formation in 2000.

(a) At what rate were the gross annual earnings of the company growing with respect to time in 2004?

(b) At what percentage rate were the gross annual earnings growing with respect to time in 2004?

6. Your starting salary will be 6,000 € per year, and you will get a raise of 500 € each year after your first year.

(a) Express the percentage rate of change of your salary as a function of time and draw the graph.

(b) At what percentage rate will your salary be increasing after 1 year?

(c) What will happen to the percentage rate of change of your salary in the long run (i.e., as t increases without bound)?

7. Suppose that the demand for a product depends on the price p (in euros) according to
$$q(p) = \frac{1{,}000}{\sqrt{p}} - 1.$$
Find and exlpain the meaning of the rate of change of demand with respect to price when the price is
   (a) 25 €;
   (b) 100 €.

8. Suppose that the demand for a product depends on the price p (in euros) according to
$$D(p) = \frac{50{,}000}{p^2} - \frac{1}{2}.$$
Find and exlpain the meaning of the rate of change of demand with respect to price when the price is
   (a) $p = 50$;
   (b) $p = 100$.

9. Suppose that for a certain city the cost of obtaining dirnking water that contains $p\%$ impurities (by volume) is given by
$$C(p) = \frac{120{,}000}{p} - 1{,}200.$$
   (a) Find the rate of change of cost with respect to p when impurities account for 1%.
   (b) Write a sentance that explains the meaning of your answer in (a).

10. The price of a certain product when the demand is q units is given by
$$p(q) = 50 - 0.03q^2.$$

(a) Find the q- and q-intercepts for this function and interpret them in terms of demand and price for this product.

(b) Find $p(20)$, give units and interpret the result in terms of demand and price for the product.

(c) Find $p'(20)$, give units and interpret the result in terms of demand and price for the product.