2.4 Marginal Analysis: Approximation by Increments

Exercises

1. A manufacturer's total cost is $C(q) = 0.1q^3 - 0.5q^2 + 500q + 200$ euros, where q is the number of units produced.

   (a) Use marginal analysis to estimate the cost of manufacturing the ninth unit.

   (b) Compute the actual cost of manufacturing the ninth unit.

2. A manufacturer's total monthly revenue is $R(q) = 240q + 0.05q^2$ euros when q units are produced and sold during the month. Currently, the manufacturer is producing 80 units a month and is planning to increase the monthly output by 1 unit.

   (a) Use marginal analysis to estimate the additional revenue that will be generated by the production and sale of the 81-st unit.

   (b) Use the revenue function to compute the actual additional revenue that will be generated by the production and sale of the 81-st unit.

In Exercises 3–8, $C(x)$ is the total cost of producing x units of a particular commodity and $p(x)$ is the price at which all x units will be sold.

   (a) Find the marginal cost and the marginal revenue.

   (b) Use marginal cost to estimate the cost of producing the fourth unit.

   (c) Find the actual cost of producing the fourth unit.

   (d) Use marginal revenue to estimate the revenue derived from the sale of the fourth unit.

   (e) Find the actual revenue derived from the sale of the fourth unit.

3. $C(x) = \frac{1}{8}x^2 + 3x + 98$, $p(x) = 25 - \frac{1}{3}x$.

4. $C(x) = \frac{1}{4}x^2 + 3x + 67$, $p(x) = \frac{1}{5}(45 - x)$.

5. $C(x) = \frac{1}{3}x^2 + 2x + 39$, $p(x) = -x^2 + 4x + 10$.

6. $C(x) = \frac{5}{9}x^2 + 5x + 73$, $p(x) = -x^2 + 2x + 13$.

7. $C(x) = \frac{1}{4}x^2 + 43$, $p(x) = \frac{3+2x}{1+x}$.

8. $C(x) = \frac{2}{7}x^2 + 65$, $p(x) = \frac{12+2x}{3+x}$.

9. The price of a certain product in a competitive market is 300 €. Suppose that the fixed cost of producing the product is 200 € and the variable cost per unit is $2 + x$ euros, where x is the number of units produced, and that the total cost function is $C(x) = 200 + 2x + x^2$.

   (a) Find the profit from the production and sale 20 units.

   (b) Find the marginal profit function.

   (c) Find the marginal profit when 20 units are produced and sold.

   (d) Estimate the additional profit generated by the production and sale of the 21-st unit.

   (e) Compute the actual additional profit from the production and sale of the 21-st unit.

10. Estimate how much the function $f(x) = x^2 - 3x + 5$ will change as x increases from 5 to 5.3.

11. Estimate how much the function $f(x) = \frac{x}{x+1} - 3$ will change as x decreases from 4 to 3.8.

12. A manufacturer's total cost is $C(q) = 0.1q^3 - 0.5q^2 + 500q + 200$ euros when the level of production is q units. The current level of production is 4 units, and the manufacturer is planning to increase this to 4.1 units. Estimate how the total cost will change as a result.

13. A manufacturer's total monthly revenue is $R(q) = 240q + 0.05q^2$ euros when q units are sold during the month. Currently, the manufacturer is producing 80 units a month and is planning to decrease the monthly output by 0.65 unit. Estimate how the total monthly revenue will change as a result.

14. The daily output of a certain factory is $Q(L) = 300L^{2/3}$ units, where L denotes the size of the labor force measured in worker-hours. Currently, 512 worker-hours of labor are used each day. Estimate the number of additional worker hours of labor that will be needed to increase daily output by 12.5 units.

15. A manufacturer's total cost is $C(q) = \frac{1}{6}q^3 + 642q + 400$ euros when q units are produced. The current level of production is 4 units. Estimate the amount by which the manufacturer should decrease the production to reduce the total cost by 130 €.

16. An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 will have assembled

$$f(t) = -t^3 + 6t^2 + 15t$$

radio-transistors t hours later. Approximately how many radios will the worker assemble between 9:00 and 9:15?

17. At a certain factory, the daily output is $Q(K) = 600\sqrt{K}$ units, where K denotes the capital investment measured in units of 1,000 €. The current capital investment is 900,000 €. Estimate the effect that an additional capital investment of 800 €will have on the daily output.

18. A manufacturer's total cost is $C(q) = 0.5q^2 + 500q + 200$ euros when the production level is q units.

    (a) Find the average cost and the marginal average cost for the commodity.

    (b) For what level of production is marginal average cost equal to 0?

    (c) For what level of production does marginal cost equal average cost?