2.5 The Chain Rule

Exercises

1. Compute the derivative $\frac{dy}{dx}$ if
   (a) $y = u^2 + 1$, $u = 3x - 2$;
   (b) $y = 2u^2 - u + 5$, $u = 1 - x^2$;
   (c) $y = \sqrt{u}$, $u = x^2 + 2x - 3$;
   (d) $y = u^2 + 2u - 3$, $u = \sqrt{x}$;
   (e) $y = \frac{1}{u^2}$, $u = x^2 + 1$;
   (f) $y = \frac{1}{\sqrt{u}}$, $u = x^2 - 9$;
   (g) $y = u^2 + u - 2$, $u = \frac{1}{x}$;
   (h) $y = \frac{1}{u-1}$, $u = x^2$;
   (i) $y = u^2$, $u = \frac{1}{x-1}$.
2. Differentiate the following functions:
   (a) $f(x) = (2x + 1)^4$;
   (b) $f(x) = \sqrt{5x^6 - 12}$;
   (c) $f(x) = (x^5 - 4x^3 - 7)^8$;
   (d) $f(t) = (3t^4 - 7t^2 + 9)^5$;
   (e) $f(t) = \frac{1}{5t^2-6t+2}$;
   (f) $g(x) = \frac{1}{4x^2+1}$;
   (g) $h(t) = (1 + \sqrt{3t})^5$;
   (h) $f(x) = \sqrt{\frac{3x+1}{2x-1}}$.
3. The gross annual earning of a certain company are $A(t) = \sqrt{10t^2 + t + 236}$ thousand euros t years after its formation in 2008.
   (a) At what rate with respect to time will the gross annual earnings of the company be growing in 2013?

(b) At what percentage rate with respect to time will the gross annual earnings of the company be growing in 2013?

4. At a certain factory, the total cost of manufacturing q units during the daily production run is $C(q) = \frac{1}{3}q^2 + 4q + 53$ euros. From the experience it has been determined that approximately $q(t) = 0.2t^2 + 0.03t$ units are manufactured during the first t hours of a production run. Compute the rate at which the total manufacturing cost is changing with respect to time 4 hours after production commences?

5. An importer of Brazilian coffee estimates that local consumers will buy approximately $D(p) = \frac{4{,}374}{p^2}$ kilograms of coffee per week when the price is p euros per kilogram. It is also estimated that t weeks from now, the price of Brazilian coffee will be $p(t) = 0.02t^2 + 0.1t + 6$ euros për kilogram. At what rate will the weekly demand for the coffee be changing with respect to time 10 weeks from now? Will the demand be increasing or decreasing?

6. When a certain commodity is sold for p euros per unit, consumers will buy $D(p) = \frac{8{,}000}{p}$ units per month. It is estimated that t months from now the price of the commodity will be $p(t) = 0.04t^{3/2} + 15$ euros per unit. At what percentage rate will the monthly demand for the commodity be changing with respect to time 25 months from now? Will the demand be increasing or decreasing?

7. When a certain commodity is sold for p euros per unit, consumers will buy $D(p) = \frac{40{,}000}{p}$ units per month. It is estimated that t months from now the price of the commodity will be $p(t) = 0.04t^{3/2} + 6.8$ euros per unit. At what percentage rate will the monthly demand for the commodity be changing with respect to time 4 months from now?

In Exercises 8–11, $C(x)$ is the total cost of producing x units of a particular commodity and $p(x)$ is the price at which all x units will be sold. Find

(a) the marginal cost;

(b) the average cost and the marginal average cost;

(c) the total revenue and the marginal revenue;

(d) the level of production x where marginal revenue equals the marginal cost;

(e) the level of production x where marginal cost equals the average cost.

8. $C(x) = e^{0.2x}$, $p(x) = e^{-3x}$.

9. $C(x) = x^3 + 20$, $p(x) = 2e^{-2x}$.

10. $C(x) = x^2 + 2$, $p(x) = \frac{\ln(x+3)}{x+3}$.

11. $C(x) = 9x + 5xe^{-2x}$, $p(x) = e^{-x}$.