2.6 The Second Derivative. Increasing and Decreasing Functions

Exercises

1. Find the second derivative of the given function:

   (a) $f(x) = 5x^{10} - 6x^5 - 27x + 4$;

   (b) $f(x) = \frac{2}{5}x^5 - 4x^3 + 9x^2 - 6x - 2$;

   (c) $y = 5\sqrt{x} + \frac{3}{x^2} + \frac{1}{3\sqrt{x}} + \frac{1}{2}$;

   (d) $y = \frac{3}{2x^5} - \sqrt{2x} + \sqrt{2}x - \frac{1}{6\sqrt{x}}$;

   (e) $f(x) = (3x - 1)^4$;

   (f) $f(x) = (x^2 + 1)^5$;

   (g) $f(t) = \frac{2}{5t+1}$;

   (h) $f(x) = \frac{3x-2}{(x-1)^2}$.

2. An efficiency study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8:00 will have produced $Q(t) = -t^3 + 8t^2 + 15t$ units t hours later.

   (a) Compute the worker's rate of production at 9:00?

   (b) At what rate is the worker's rate of production changing with respect to time at 9:00?

   (c) Use calculus to estimate the change in the worker's rate of production between 9:00 and 9:15.

   (d) Compute the actual change in the worker's rate of production between 9:00 and 9:15.

3. It is projected that t years from now, the average price per unit in a certain sector of the economy will be $p(t) = -t^3 + 7t^2 + 200t + 300$ euros.

   (a) At what rate will the price per unit be increasing with respect to time 5 years from now?

(b) At what rate will change the rate of price increase with respect to time 5 years from now?

(c) Use calculus to estimate the change in the rate of price increase during the first half of the sixth year.

4. Find the intervals of increase and decrease for the given functions:

   (a) $f(x) = x^2 - 4x + 5$;

   (b) $f(t) = t^3 + 3t^2 + 1$;

   (c) $f(x) = 2x^3 + 3x^2 - 12x - 7$.

5. Find the critical points and classify each as a relative maximum, relative minimum or neither for the following functions:

   (a) $f(x) = 3x^4 - 8x^3 + 6x^2 + 2$;

   (b) $f(x) = 4x^3 - 72x^2 + 324x$;

   (c) $f(t) = 2t^3 + 6t^2 + 6t + 5$;

   (d) $f(x) = (x-1)^5$;

   (e) $f(x) = \frac{x^2}{x-1}$.

6. The total cost of producing x units of a certain commodity is $C(x) = \sqrt{5x+2} + 3$ euros. Sketch the cost curve and find the marginal cost. Does marginal cost increase or decrease with increasing production?

7. Let $p(x) = (10-3x)^2$ be the price at which x units of a certain commodity will be sold. Sketch the curves the revenue and marginal revenue curves on the same graph. For what level of production is revenue maximized?

8. To produce x units of a particular commodity, a monopolist has a total cost of $C(x) = 2x^2 + 3x + 5$ and $p(x) = 5 - 2x$ is the price at which all the x units will be sold. Find the profit function and sketch its graph. For what level of production does the profit appear to be maximized?

9. An efficiency study of the morning shift (from 8:00 to 12:00) at a certain factory indicates that an average worker who arrives on the job at 8:00 will have produced $Q(t) = -t^3 + \frac{9}{2}t^2 + 15t$ units t hours later.

(a) At what time during the morning is the worker performing most efficiently?

(b) At what time during the morning is the worker performing lease efficiently?

10. If the total cost function for a commodity is given by $C(x) = \frac{1}{4}x^2 + 4x + 100$ when x units are produced, producing how many units will result in a minimum average cost per unit? Find the minimum average cost.

11. A firm in a competitive market must sell its product 200 € per unit. The average cost per unit is $\bar{C}(x) = 80 + x$, where x is the number of units produced and sold. How many units should be sold to maximize the profit? Find the maximum profit.

12. A cable TV and Internet provider company has 1,000 costumers paying 20 € each month. If each 1 € reduction in price attracts 100 new costumers, find the price that yields maximum revenue. Find the maximum revenue.

13. If organizers charge 5 € admission to a show, 1,000 people will attend, and for each 1 € increase in price, 100 fewer people will attend. What price will give the maximum revenue for the show? Find the maximum revenue.