CHAPTER 5

Discrete Random Variables

Learning Objectives

After mastering the material in this chapter, you will be able to:

- **LO5-1** Explain the difference between a discrete random variable and a continuous random variable.
- **LO5-2** Find a discrete probability distribution and compute its mean and standard deviation.
- **LO5-3** Use the binomial distribution to compute probabilities.
- **LO5-4** Use the Poisson distribution to compute probabilities (Optional).
- **LO5-5** Use the hypergeometric distribution to compute probabilities (Optional).
- **LO5-6** Compute and understand the covariance between two random variables (Optional).

Chapter Outline

We often use what we call **random variables** to describe the important aspects of the outcomes of experiments. In this chapter we introduce two important types of random variables—**discrete random variables** and **continuous random variables**—and learn how to find probabilities concerning discrete random variables. As one application, we will begin to see how to use probabilities concerning discrete random variables to make statistical inferences about populations.

5.1 Two Types of Random Variables

LO5-1 Explain the difference between a discrete random variable and a continuous random variable.

We begin with the definition of a random variable:

A **random variable** is a variable that assumes numerical values that are determined by the outcome of an experiment, where one and only one numerical value is assigned to each experimental outcome.

Before an experiment is carried out, its outcome is uncertain. It follows that, because a random variable assigns a number to each experimental outcome, a random variable can be thought of as *representing an uncertain numerical outcome.*

To illustrate the idea of a random variable, suppose that Sound City sells and installs car stereo systems. One of Sound City's most popular stereo systems is the TrueSound-XL, a top-of-the-line satellite car radio. Consider (the experiment of) selling the TrueSound-XL radio at the Sound City store during a particular week. If we let x denote the number of radios sold during the week, then x is a random variable. That is, looked at before the week, the number of radios x that will be sold is uncertain, and, therefore, x is a random variable.

Notice that x, the number of TrueSound-XL radios sold in a week, might be 0 or 1 or 2 or 3, and so forth. In general, when the possible values of a random variable can be counted or listed, we say that the random variable is a **discrete random variable.** That is, either a discrete random variable may assume a finite number of possible values or the possible values may take the form of a *countable* sequence or list such as 0, 1, 2, 3, 4, . . . (a *countably infinite* list).

Some other examples of discrete random variables are

1. The number, x, of the next three customers entering a store who will make a purchase. Here x could be 0, 1, 2, or 3.
2. The number, x, of four patients taking a new antibiotic who experience gastrointestinal distress as a side effect. Here x could be 0, 1, 2, 3, or 4.
3. The number, x, of television sets in a sample of 8 five-year-old television sets that have not needed a single repair. Here x could be any of the values 0, 1, 2, 3, 4, 5, 6, 7, or 8.
4. The number, x, of major fires in a large city in the next two months. Here x could be 0, 1, 2, 3, and so forth (there is no definite maximum number of fires).
5. The number, x, of dirt specks in a one-square-yard sheet of plastic wrap. Here x could be 0, 1, 2, 3, and so forth (there is no definite maximum number of dirt specks).

The values of the random variables described in examples 1, 2, and 3 are countable and finite. In contrast, the values of the random variables described in 4 and 5 are countable and infinite (or countably infinite lists). For example, in theory there is no limit to the number of major fires that could occur in a city in two months.

Not all random variables have values that are countable. When a random variable may assume any numerical value in one or more intervals on the real number line, then we say that the random variable is a **continuous random variable.**

EXAMPLE 5.1 The Car Mileage Case: A Continuous Random Variable

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Consider the car mileage situation that we have discussed in Chapters 1–3. The EPA combined city and highway mileage, x, of a randomly selected midsize car is a continuous random variable. This is because, although we have measured mileages to the nearest one-tenth of a mile per gallon, technically speaking, the potential mileages that might be obtained correspond (starting at, perhaps, 26 mpg) to an interval of numbers on the real line. We cannot count or list the numbers in such an interval because they are infinitesimally close together. That is, given any two numbers in an interval on the real line, there is always another number between them. To understand this, try listing the mileages starting with 26 mpg. Would the next mileage be 26.1 mpg? No, because we could obtain a mileage of 26.05 mpg. Would 26.05 mpg be the next mileage? No, because we could obtain a mileage of 26.025 mpg. We could continue this line of reasoning indefinitely. That is, whatever value we would try to list as the *next mileage,* there would always be another mileage between this *next mileage* and 26 mpg.

Some other examples of continuous random variables are

1. The temperature (in degrees Fahrenheit) of a cup of coffee served at a McDonald's restaurant.
2. The weight (in ounces) of strawberry preserves dispensed by an automatic filling machine into a 16-ounce jar.
3. The time (in minutes) that a customer in a store must wait to receive a credit card authorization.
4. The interest rate (in percent) charged for mortgage loans at a bank.

Exercises for Section 5.1

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CONCEPTS

5.1 Explain the concept of a random variable.

5.2 Explain how the values of a discrete random variable differ from the values of a continuous random variable.

5.3 Classify each of the following random variables as discrete or continuous:

a x = the number of girls born to a couple who will have three children.
b x = the number of defects found on an automobile at final inspection.
c x = the weight (in ounces) of the sandwich meat placed on a submarine sandwich.
d x = the number of incorrect lab procedures conducted at a hospital during a particular week.
e x = the number of customers served during a given day at a drive-through window.
f x = the time needed by a clerk to complete a task.
g x = the temperature of a pizza oven at a particular time.

5.2 Discrete Probability Distributions

LO5-2 Find a discrete probability distribution and compute its mean and standard deviation.

The value assumed by a discrete random variable depends on the outcome of an experiment. Because the outcome of the experiment will be uncertain, the value assumed by the random variable will also be uncertain. However, it is often useful to know the probabilities that are associated with the different values that the random variable can take on. That is, we often wish to know the random variable's **probability distribution.**

The **probability distribution** of a discrete random variable is a table, graph, or formula that gives the probability associated with each possible value that the random variable can assume.

We denote the probability distribution of the discrete random variable x as $p(x)$. As we will demonstrate in Section 5.3 (which discusses the *binomial distribution*), we can sometimes use the sample space of an experiment and probability rules to find the probability distribution of a random variable. In other situations we collect data that will allow us to estimate the probabilities in a probability distribution.

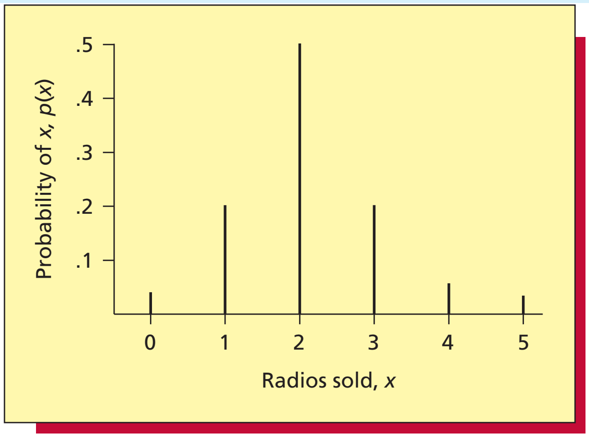
EXAMPLE 5.2 The Sound City Case: Selling TrueSound-XL Radios

Recall that Sound City sells the TrueSound-XL car radio, and define the random variable x to be the number of such radios sold in a particular week at Sound City. Also, suppose that Sound City has kept historical records of TrueSound-XL sales during the last 100 weeks. These records show

TABLE 5.1 An Estimate (Based on 100 Weeks of Historical Data) of the Probability Distribution of *x*, the Number of TrueSound-XL Radios Sold at Sound City in a Week

x, Number of Radios Sold	*p*(*x*), the Probability of *x*
0	$p(0) = P(x = 0) = 3/100 = .03$
1	$p(1) = P(x = 1) = 20/100 = .20$
2	$p(2) = P(x = 2) = 50/100 = .50$
3	$p(3) = P(x = 3) = 20/100 = .20$
4	$p(4) = P(x = 4) = 5/100 = .05$
5	$p(5) = P(x = 5) = 2/100 = .02$

FIGURE 5.1 A Graph of the Probability Distribution of *x*, the Number of TrueSound-XL Radios Sold at Sound City in a Week



3 weeks with no radios sold, 20 weeks with one radio sold, 50 weeks with two radios sold, 20 weeks with three radios sold, 5 weeks with four radios sold, and 2 weeks with five radios sold. Because the records show 3 of 100 weeks with no radios sold, we estimate that $p(0) = p(x = 0)$ is $3/100 = .03$. That is, we estimate that the probability of no radios being sold during a week is .03. Similarly, because the records show 20 of 100 weeks with one radio sold, we estimate that $p(1) = P(x = 1)$ is $20/100 = .20$. That is, we estimate that the probability of exactly one radio being sold during a week is .20. Continuing in this way for the other values of the random variable x, we can estimate $p(2) = P(x = 2)$, $p(3) = P(x = 3)$, $p(4) = P(x = 4)$, and $p(5) = P(x = 5)$. Table 5.1 gives the entire estimated probability distribution of the number of TrueSound-XL radios sold at Sound City during a week, and Figure 5.1 shows a graph of this distribution. Moreover, such a probability distribution helps us to more easily calculate probabilities about events related to a random variable. For example, the probability of at least two radios being sold at Sound City during a week [that is, $P(x \geq 2)$] is $p(2) + p(3) + p(4) + p(5) = .50 + .20 + .05 + .02 = .77$. This says that we estimate that in 77 percent of all weeks, at least two TrueSound-XL radios will be sold at Sound City.

Finally, note that using historical sales data to obtain the estimated probabilities in Table 5.1 is reasonable if the TrueSound-XL radio sales process is stable over time. This means that the number of radios sold weekly does not exhibit any long-term upward or downward trends and is not seasonal (that is, radio sales are not higher at one time of the year than at others).

In general, a discrete probability distribution $p(x)$ must satisfy two conditions:

Properties of a Discrete Probability Distribution *p*(*x*)

A **discrete probability distribution *p*(*x*)** must be such that

1. $p(x) \geq 0$ for each value of x
2. $\sum_{\text{All } x} p(x) = 1$

The first of these conditions says that each probability in a probability distribution must be zero or positive. The second condition says that the probabilities in a probability distribution must sum to 1. Looking at the probability distribution illustrated in Table 5.1, we can see that these properties are satisfied.

Suppose that the experiment described by a random variable x is repeated an indefinitely large number of times. If the values of the random variable x observed on the repetitions are recorded, we would obtain the population of all possible observed values of the random variable x. This population has a mean, which we denote as μ_x and which we sometimes call the **expected value of *x*.** In order to calculate μ_x, we multiply each value of x by its probability $p(x)$ and then sum the resulting products over all possible values of x.

The Mean, or Expected Value, of a Discrete Random Variable

The **mean,** or **expected value,** of a discrete random variable x is

$$\mu_x = \sum_{\text{All } x} x p(x)$$

EXAMPLE 5.3 The Sound City Case: Selling TrueSound-XL Radios

Remember that Table 5.1 gives the probability distribution of x, the number of TrueSound-XL radios sold in a week at Sound City. Using this distribution, it follows that

$$\begin{aligned} \mu_x &= \sum_{\text{All } x} x p(x) \\ &= 0p(0) + 1p(1) + 2p(2) + 3p(3) + 4p(4) + 5p(5) \\ &= 0(.03) + 1(.20) + 2(.50) + 3(.20) + 4(.05) + 5(.02) \\ &= 2.1 \end{aligned}$$

To see that such a calculation gives the mean of all possible observed values of x, recall from Example 5.2 that the probability distribution in Table 5.1 was estimated from historical records of TrueSound-XL sales during the last 100 weeks. Also recall that these historical records tell us that during the last 100 weeks Sound City sold

1. Zero radios in 3 of the 100 weeks, for a total of $0(3) = 0$ radios
2. One radio in 20 of the 100 weeks, for a total of $1(20) = 20$ radios
3. Two radios in 50 of the 100 weeks, for a total of $2(50) = 100$ radios
4. Three radios in 20 of the 100 weeks, for a total of $3(20) = 60$ radios
5. Four radios in 5 of the 100 weeks, for a total of $4(5) = 20$ radios
6. Five radios in 2 of the 100 weeks, for a total of $5(2) = 10$ radios

In other words, Sound City sold a total of

$$0 + 20 + 100 + 60 + 20 + 10 = 210 \text{ radios}$$

in 100 weeks, or an average of $210/100 = 2.1$ radios per week. Now, the average

$$\frac{210}{100} = \frac{0 + 20 + 100 + 60 + 20 + 10}{100}$$

can be written as

$$\frac{0(3) + 1(20) + 2(50) + 3(20) + 4(5) + 5(2)}{100}$$

which can be rewritten as

$$\begin{aligned} &0\left(\frac{3}{100}\right) + 1\left(\frac{20}{100}\right) + 2\left(\frac{50}{100}\right) + 3\left(\frac{20}{100}\right) + 4\left(\frac{5}{100}\right) + 5\left(\frac{2}{100}\right) \\ &= 0(.03) + 1(.20) + 2(.50) + 3(.20) + 4(.05) + 5(.02) \end{aligned}$$

which is $\mu_x = 2.1$. That is, if observed sales values occur with relative frequencies equal to those specified by the probability distribution in Table 5.1, then the average number of radios sold per week is equal to the expected value of x and is 2.1 radios.

Of course, if we observe radio sales for another 100 weeks, the relative frequencies of the observed sales values would not (unless we are very lucky) be exactly as specified by the estimated probabilities in Table 5.1. Rather, the observed relative frequencies would differ somewhat from the estimated probabilities in Table 5.1, and the average number of radios sold per week would not exactly equal $\mu_x = 2.1$ (although the average would likely be close). However, the point is this: If the probability distribution in Table 5.1 were the true probability distribution of weekly radio sales, and if we were to observe radio sales for an indefinitely large number of weeks, then we would observe sales values with relative frequencies that are exactly equal to those specified by the probabilities in Table 5.1. In this case, when we calculate the expected value of x to be $\mu_x = 2.1$, we are saying that *in the long run* (that is, over an indefinitely large number of weeks) Sound City would average selling 2.1 TrueSound-XL radios per week.

EXAMPLE 5.4 The Life Insurance Case: Setting a Policy Premium

An insurance company sells a $20,000 whole life insurance policy for an annual premium of $300. Actuarial tables show that a person who would be sold such a policy with this premium has a .001 probability of death during a year. Let x be a random variable representing the insurance company's profit made on one of these policies during a year. The probability distribution of x is

***x*, Profit**	***p*(*x*), Probability of *x***
$300 (if the policyholder lives)	.999
$300 − $20,000 = −$19,700 (a $19,700 loss if the policyholder dies)	.001

The expected value of x (expected profit per year) is

$$\begin{aligned}\mu_x &= \$300(.999) + (-\$19{,}700)(.001)\\ &= \$280\end{aligned}$$

This says that if the insurance company sells a very large number of these policies, it will average a profit of $280 per policy per year. Because insurance companies actually do sell large numbers of policies, it is reasonable for these companies to make profitability decisions based on expected values.

Next, suppose that we wish to find the premium that the insurance company must charge for a $20,000 policy if the company wishes the average profit per policy per year to be greater than $0. If we let *prem* denote the premium the company will charge, then the probability distribution of the company's yearly profit x is

***x*, Profit**	***p*(*x*), Probability of *x***
prem (if policyholder lives)	.999
prem − $20,000 (if policyholder dies)	.001

The expected value of x (expected profit per year) is

$$\begin{aligned}\mu_x &= prem(.999) + (prem - 20{,}000)(.001)\\ &= prem - 20\end{aligned}$$

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In order for this expected profit to be greater than zero, the premium must be greater than $20. If, as previously stated, the company charges $300 for such a policy, the $280 charged in excess of the needed $20 compensates the company for commissions paid to salespeople, administrative costs, dividends paid to investors, and other expenses.

In general, it is reasonable to base decisions on an expected value if we perform the experiment related to the decision (for example, if we sell the life insurance policy) many times. If we do not (for instance, if we perform the experiment only once), then it may not be a good idea to base decisions on the expected value. For example, it might not be wise for you—as an individual—to sell one person a $20,000 life insurance policy for a premium of $300. To see this, again consider the probability distribution of yearly profit:

***x*, Profit**	***p*(*x*), Probability of *x***
\$300 (if policyholder lives)	.999
\$300 − \$20,000 = −\$19,700 (if policyholder dies)	.001

and recall that the expected profit per year is \$280. However, because you are selling only one policy, you will not receive the \$280. You will either gain \$300 (with probability .999) or you will lose \$19,700 (with probability .001). Although the decision is personal, and although the chance of losing \$19,700 is very small, many people would not risk such a loss when the potential gain is only \$300.

Just as the population of all possible observed values of a discrete random variable x has a mean μ_x, this population also has a variance σ_x^2 and a standard deviation σ_x. Recall that the variance of a population is the average of the squared deviations of the different population values from the population mean. To find σ_x^2, we calculate $(x - \mu_x)^2$ for each value of x, multiply $(x - \mu_x)^2$ by the probability $p(x)$, and sum the resulting products over all possible values of x.

The Variance and Standard Deviation of a Discrete Random Variable

The **variance** of a discrete random variable x is

$$\sigma_x^2 = \sum_{\text{All } x} (x - \mu_x)^2 p(x)$$

The **standard deviation** of x is the positive square root of the variance of x. That is,

$$\sigma_x = \sqrt{\sigma_x^2}$$

EXAMPLE 5.5 The Sound City Case: Selling TrueSound-XL Radios

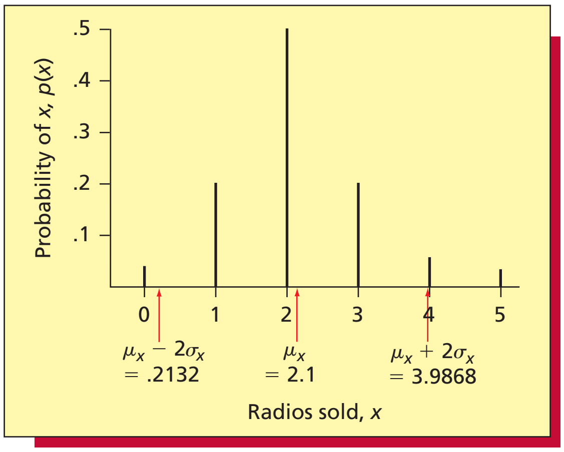
Table 5.1 gives the probability distribution of x, the number of TrueSound-XL radios sold in a week at Sound City. Remembering that we have calculated μ_x (in Example 5.3) to be 2.1 radios, it follows that

$$\begin{aligned}\sigma_x^2 &= \sum_{\text{All } x} (x - \mu_x)^2 p(x) \\ &= (0 - 2.1)^2 p(0) + (1 - 2.1)^2 p(1) + (2 - 2.1)^2 p(2) + (3 - 2.1)^2 p(3) \\ &\quad + (4 - 2.1)^2 p(4) + (5 - 2.1)^2 p(5) \\ &= (4.41)(.03) + (1.21)(.20) + (.01)(.50) + (.81)(.20) + (3.61)(.05) + (8.41)(.02) \\ &= .89\end{aligned}$$

and that the standard deviation of x is $\sigma_x = \sqrt{.89} = .9434$ radios. To make one interpretation of a standard deviation of .9434 radios, suppose that Sound City sells another top-of-the-line satellite car radio called the ClearTone-400. If the ClearTone-400 also has mean weekly sales of 2.1 radios, and if the standard deviation of the ClearTone-400's weekly sales is 1.2254 radios, we would conclude that there is more variability in the weekly sales of the ClearTone-400 than in the weekly sales of the TrueSound-XL.

In Chapter 3 we considered the percentage of measurements in a population that are within (plus or minus) one, two, or three standard deviations of the mean of the population. Similarly, we can consider the probability that a random variable x will be within (plus or minus) one, two,

FIGURE 5.2 **The Interval $[\mu_x \pm 2\sigma_x]$ for the Probability Distribution Describing TrueSound-XL Radio Sales (see Table 5.1)**



or three standard deviations of the mean of the random variable. For example, consider the probability distribution in Table 5.1 of x, the number of TrueSound-XL radios sold in a week at Sound City. Also, recall that $\mu_x = 2.1$ and $\sigma_x = .9434$. If (for instance) we wish to find the probability that x will be within (plus or minus) two standard deviations of μ_x, then we need to find the probability that x will lie in the interval

$$\begin{aligned}[\mu_x \pm 2\sigma_x] &= [2.1 \pm 2(.9434)] \\ &= [.2132,\ 3.9868]\end{aligned}$$

As illustrated in Figure 5.2, there are three values of x ($x = 1$, $x = 2$, and $x = 3$) that lie in the interval [.2132, 3.9868]. Therefore, the probability that x will lie in the interval [.2132, 3.9868] is the probability that x will equal 1 or 2 or 3, which is $p(1) + p(2) + p(3) = .20 + .50 + .20 = .90$. This says that in 90 percent of all weeks, the number of TrueSound-XL radios sold at Sound City will be within (plus or minus) two standard deviations of the mean weekly sales of the TrueSound-XL radio at Sound City.

In general, consider any random variable with mean μ_x and standard deviation σ_x. Then, Chebyshev's Theorem (see Chapter 3, page 116) tells us that, for any value of k that is greater than 1, the probability is at least $1 - 1/k^2$ that x will be within (plus or minus) k standard deviations of μ_x and thus will lie in the interval $[\mu_x \pm k\sigma_x]$. For example, setting k equal to 2, the probability is at least $1 - 1/2^2 = 1 - 1/4 = 3/4$ that x will lie in the interval $[\mu_x \pm 2\sigma_x]$. Setting k equal to 3, the probability is at least $1 - 1/3^2 = 1 - 1/9 = 8/9$ that x will lie in the interval $[\mu_x \pm 3\sigma_x]$. If (as in the Sound City situation) we have the probability distribution of x, we can calculate exact probabilities, and thus we do not need the approximate probabilities given by Chebyshev's Theorem. However, in some situations we know the values of μ_x and σ_x, but we do not have the probability distribution of x. In such situations the approximate Chebyshev's probabilities can be quite useful. For example, let x be a random variable representing the return on a particular investment, and suppose that an investment prospectus tells us that, based on historical data and current market trends, the investment return has a mean (or expected value) of $\mu_x = \$1,000$ and a standard deviation of $\sigma_x = \$100$. It then follows from Chebyshev's Theorem that the probability is at least 8/9 that the investment return will lie in the interval $[\mu_x \pm 3\sigma_x] = [1000 \pm 3(100)] = [700, 1300]$. That is, the probability is fairly high that the investment will have a minimum return of \$700 and a maximum return of \$1300.

In the next several sections, we will see that a probability distribution $p(x)$ is sometimes specified by using a formula. As a simple example of this, suppose that a random variable x is equally likely to assume any one of n possible values. In this case we say that x is described by the discrete **uniform distribution** and we specify $p(x)$ by using the formula $p(x) = 1/n$. For example, if we roll a fair die and x denotes the number of spots that show on the upward face of the die, then x is uniformly distributed and $p(x) = 1/6$ for $x = 1, 2, 3, 4, 5$, and 6. As another example, if historical sales records show that a Chevrolet dealership is equally likely to sell 0, 1, 2, or 3 Chevy Malibus in a given week, and if x denotes the number of Chevy Malibus that the dealership sells in a week, then x is uniformly distributed and $p(x) = 1/4$ for $x = 0, 1, 2$, and 3.

Exercises for Section 5.2

CONCEPTS

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5.4 What is a discrete probability distribution? Explain in your own words.

5.5 What conditions must be satisfied by the probabilities in a discrete probability distribution? Explain what these conditions mean.

5.6 Describe how to compute the mean (or expected value) of a discrete random variable, and interpret what this quantity tells us about the observed values of the random variable.

5.7 Describe how to compute the standard deviation of a discrete random variable, and interpret what this quantity tells us about the observed values of the random variable.

METHODS AND APPLICATIONS

5.8 Recall from Example 5.5 that Sound City also sells the ClearTone-400 satellite car radio. For this radio, historical sales records over the last 100 weeks show 6 weeks with no radios sold, 30 weeks with one radio sold, 30 weeks with two radios sold, 20 weeks with three radios sold, 10 weeks with four radios sold, and 4 weeks with five radios sold. Estimate and write out the probability distribution of x, the number of ClearTone-400 radios sold at Sound City during a week.

5.9 Use the estimated probability distribution in Exercise 5.8 to calculate μ_x, σ_x^2, and σ_x.

5.10 Use your answers to Exercises 5.8 and 5.9 to calculate the probabilities that x will lie in the intervals $[\mu_x \pm \sigma_x]$, $[\mu_x \pm 2\sigma_x]$, and $[\mu_x \pm 3\sigma_x]$.

5.11 The following table summarizes investment outcomes and corresponding probabilities for a particular oil well:

x = the outcome in \$	**$p(x)$**
−\$40,000 (no oil)	.25
10,000 (some oil)	.7
70,000 (much oil)	.05

a Graph $p(x)$; that is, graph the probability distribution of x.

b Find the expected monetary outcome. Mark this value on your graph of part *a*. Then interpret this value.

c Calculate the standard deviation of x.

5.12 In the book *Foundations of Financial Management* (7th ed.), Stanley B. Block and Geoffrey A. Hirt discuss risk measurement for investments. Block and Hirt present an investment with the possible outcomes and associated probabilities given in Table 5.2. The authors go on to say that the probabilities

> may be based on past experience, industry ratios and trends, interviews with company executives, and sophisticated simulation techniques. The probability values may be easy to determine for the introduction of a mechanical stamping process in which the manufacturer has 10 years of past data, but difficult to assess for a new product in a foreign market. DS OutcomeDist

a Use the probability distribution in Table 5.2 to calculate the expected value (mean) and the standard deviation of the investment outcomes. Interpret the expected value.

TABLE 5.2 Probability Distribution of Outcomes for an Investment DS OutcomeDist

Outcome	Probability of Outcome	Assumptions
\$300	.2	Pessimistic
600	.6	Moderately successful
900	.2	Optimistic

Source: S. B. Block and G. A. Hirt, *Foundations of Financial Management,* 7th ed., p. 378. Copyright © 1994. Reprinted by permission of McGraw-Hill Companies, Inc.

b Block and Hirt interpret the standard deviation of the investment outcomes as follows: "Generally, the larger the standard deviation (or spread of outcomes), the greater is the risk." Explain why this makes sense. Use Chebyshev's Theorem to illustrate your point.

c Block and Hirt compare three investments having the following means and standard deviations of the investment outcomes:

Investment 1	Investment 2	Investment 3
$\mu = \$600$	$\mu = \$600$	$\mu = \$600$
$\sigma = \$20$	$\sigma = \$190$	$\sigma = \$300$

Which of these investments involves the most risk? The least risk? Explain why by using Chebyshev's Theorem to compute an interval for each investment that will contain at least 8/9 of the investment outcomes.

d Block and Hirt continue by comparing two more investments:

Investment *A*	Investment *B*
$\mu = \$6{,}000$	$\mu = \$600$
$\sigma = \$600$	$\sigma = \$190$

The authors explain that Investment *A*

> appears to have a high standard deviation, but not when related to the expected value of the distribution. A standard deviation of \$600 on an investment with an expected value of \$6,000 may indicate less risk than a standard deviation of \$190 on an investment with an expected value of only \$600.
>
> We can eliminate the size difficulty by developing a third measure, the **coefficient of variation** (V). This term calls for nothing more difficult than dividing the standard deviation of an investment by the expected value. Generally, the larger the coefficient of variation, the greater is the risk.
>
> $$\text{Coefficient of variation } (V) = \frac{\sigma}{\mu}$$

Calculate the coefficient of variation for investments *A* and *B*. Which investment carries the greater risk?

e Calculate the coefficient of variation for investments 1, 2, and 3 in part *c*. Based on the coefficient of variation, which investment involves the most risk? The least risk? Do we obtain the same results as we did by comparing standard deviations (in part *c*)? Why?

5.13 An insurance company will insure a \$50,000 diamond for its full value against theft at a premium of \$400 per year. Suppose that the probability that the diamond will be stolen is .005, and let x denote the insurance company's profit.

a Set up the probability distribution of the random variable x.

b Calculate the insurance company's expected profit.

c Find the premium that the insurance company should charge if it wants its expected profit to be \$1,000.

5.14 In the book *Foundations of Financial Management* (7th ed.), Stanley B. Block and Geoffrey A. Hirt discuss a semiconductor firm that is considering two choices: (1) expanding the production of semiconductors for sale to end users or (2) entering the highly competitive home computer market. The cost of both projects is \$60 million, but the net present value of the cash flows from sales and the risks are different.

FIGURE 5.3 A Tree Diagram of Two Project Choices

	(1) Sales	(2) Probability	(3) Present Value of Cash Flow from Sales ($ millions)	(4) Initial Cost ($ millions)	(5) Net Present Value, NPV = (3) − (4) ($ millions)
Start — *A*: Expand semiconductor capacity	High	.50	$100	$60	$40
	Moderate	.25	75	60	15
	Low	.25	40	60	(20)
Start — *B*: Enter home computer market	High	.20	$200	$60	$140
	Moderate	.50	75	60	15
	Low	.30	25	60	(35)

Source: S. B. Block and G. A. Hirt, *Foundations of Financial Management,* 7th ed., p. 387. Copyright © 1994. Reprinted by permission of McGraw-Hill Companies, Inc.

Figure 5.3 gives a tree diagram of the project choices. The tree diagram gives a probability distribution of expected sales for each project. It also gives the present value of cash flows from sales and the net present value (NPV = present value of cash flow from sales minus initial cost) corresponding to each sales alternative. Note that figures in parentheses denote losses.

a For each project choice, calculate the expected net present value.
b For each project choice, calculate the variance and standard deviation of the net present value.
c Calculate the coefficient of variation for each project choice. See Exercise 5.12*d* for a discussion of the coefficient of variation.
d Which project has the higher expected net present value?
e Which project carries the least risk? Explain.
f In your opinion, which project should be undertaken? Justify your answer.

5.15 Five thousand raffle tickets are to be sold at $10 each to benefit a local community group. The prizes, the number of each prize to be given away, and the dollar value of winnings for each prize are as follows: DS Raffle

Prize	Number to Be Given Away	Dollar Value
Automobile	1	$20,000
Entertainment center	2	3,000 each
DVD recorder	5	400 each
Gift certificate	50	20 each

If you buy one ticket, calculate your expected winnings. (Form the probability distribution of x = your dollar winnings, and remember to subtract the cost of your ticket.)

5.16 A survey conducted by a song rating service finds that the percentages of listeners *familiar* with *Poker Face* by Lady Gaga who would give the song ratings of 5, 4, 3, 2, and 1 are, respectively, 43 percent, 21 percent, 22 percent, 7 percent, and 7 percent. Assign the numerical values 1, 2, 3, 4, and 5 to the (qualitative) ratings 1, 2, 3, 4, and 5 and find an estimate of the probability distribution of x = this song's rating by a randomly selected listener who is familiar with the song.

5.17 In Exercise 5.16,

a Find the expected value of the estimated probability distribution.
b Interpret the meaning of this expected value in terms of all possible *Poker Face* listeners.

5.3 The Binomial Distribution

In this section we discuss what is perhaps the most important discrete probability distribution—the binomial distribution. We begin with an example.

LO5-3 Use the binomial distribution to compute probabilities.

EXAMPLE 5.6 Purchases at a Discount Store

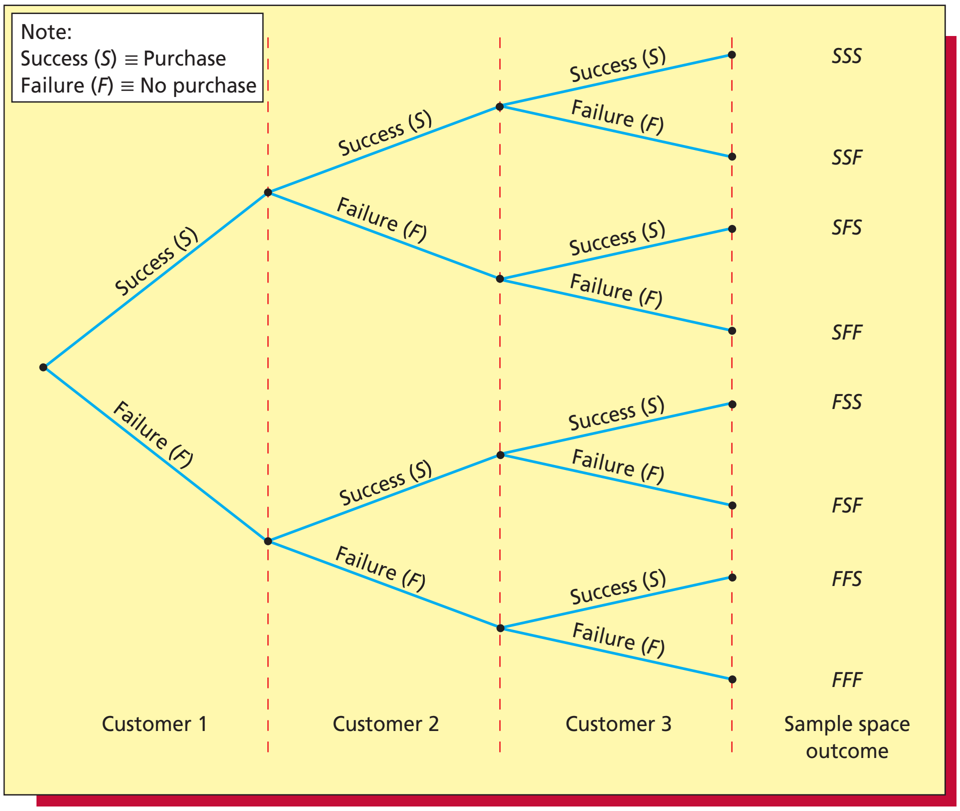
Suppose that historical sales records indicate that 40 percent of all customers who enter a discount department store make a purchase. What is the probability that two of the next three customers will make a purchase?

In order to find this probability, we first note that the experiment of observing three customers making a purchase decision has several distinguishing characteristics:

1 The experiment consists of three identical *trials;* each trial consists of a customer making a purchase decision.
2 Two outcomes are possible on each trial: the customer makes a purchase (which we call a *success* and denote as S), or the customer does not make a purchase (which we call a *failure* and denote as F).
3 Because 40 percent of all customers make a purchase, it is reasonable to assume that $P(S)$, the probability that a customer will make a purchase, is .4 and is constant for all customers. This implies that $P(F)$, the probability that a customer will not make a purchase, is .6 and is constant for all customers.
4 We assume that customers make independent purchase decisions. That is, we assume that the outcomes of the three trials are independent of each other.

Using the tree diagram in Figure 5.4, we can find the sample space of the experiment of three customers making a purchase decision. As shown in the tree diagram, the sample space of the

FIGURE 5.4 A Tree Diagram of Three Customers Making a Purchase Decision



experiment consists of the following eight sample space outcomes:

SSS	FSS
SSF	FSF
SFS	FFS
SFF	FFF

Here the sample space outcome *SSS* represents the first customer making a purchase, the second customer making a purchase, and the third customer making a purchase. On the other hand, the sample space outcome *SFS* represents the first customer making a purchase, the second customer not making a purchase, and the third customer making a purchase. In addition, each sample space outcome corresponds to a specific number of customers (out of three customers) making a purchase. For example, the sample space outcome *SSS* corresponds to all three customers making a purchase. Each of the sample space outcomes *SSF*, *SFS*, and *FSS* corresponds to two out of three customers making a purchase. Each of the sample space outcomes *SFF, FSF,* and *FFS* corresponds to one out of three customers making a purchase. Finally, the sample space outcome *FFF* corresponds to none of the three customers making a purchase.

To find the probability that two out of the next three customers will make a purchase (the probability asked for at the beginning of this example), we consider the sample space outcomes *SSF*, *SFS*, and *FSS*. Because the trials (individual customer purchase decisions) are independent, we can multiply the probabilities associated with the different trial outcomes (each of which is S or F) to find the probability of a sequence of customer outcomes:

$$P(SSF) = P(S)P(S)P(F) = (.4)(.4)(.6) = (.4)^2(.6)$$
$$P(SFS) = P(S)P(F)P(S) = (.4)(.6)(.4) = (.4)^2(.6)$$
$$P(FSS) = P(F)P(S)P(S) = (.6)(.4)(.4) = (.4)^2(.6)$$

It follows that the probability that two out of the next three customers will make a purchase is

$$\begin{aligned} &P(SSF) + P(SFS) + P(FSS) \\ &\quad = (.4)^2(.6) + (.4)^2(.6) + (.4)^2(.6) \\ &\quad = 3(.4)^2(.6) = .288 \end{aligned}$$

We can now generalize the previous result and find the probability that x of the next n customers will make a purchase. Here we will assume that p is the probability that a customer will make a purchase, $q = 1 - p$ is the probability that a customer will not make a purchase, and that purchase decisions (trials) are independent. To generalize the probability that two out of the next three customers will make a purchase, which equals $3(.4)^2(.6)$, we note that

1. The 3 in this expression is the number of sample space outcomes (*SSF*, *SFS*, and *FSS*) that correspond to the event "two out of the next three customers will make a purchase." Note that this number equals the number of ways we can arrange two successes among the three trials.
2. The .4 is p, the probability that a customer will make a purchase.
3. The .6 is $q = 1 - p$, the probability that a customer will not make a purchase.

Therefore, the probability that two of the next three customers will make a purchase is

$$\left(\begin{array}{c}\text{The number of ways}\\ \text{to arrange 2 successes}\\ \text{among 3 trials}\end{array}\right) p^2 q^1$$

Now, notice that, although each of the sample space outcomes *SSF*, *SFS*, and *FSS* represents a different arrangement of the two successes among the three trials, each of these sample space outcomes consists of two successes and one failure. For this reason, the probability of each of these sample space outcomes equals $(.4)^2(.6)^1 = p^2q^1$. It follows that p is raised to a power that equals the number of successes (2) in the three trials, and q is raised to a power that equals the number of failures (1) in the three trials.

In general, each sample space outcome describing the occurrence of x successes (purchases) in n trials represents a different arrangement of x successes in n trials. However, each outcome consists of x successes and $n - x$ failures. Therefore, the probability of each sample space outcome is $p^x q^{n-x}$. It follows by analogy that the probability that x of the next n trials will be successes (purchases) is

$$\begin{pmatrix} \text{The number of ways} \\ \text{to arrange } x \text{ successes} \\ \text{among } n \text{ trials} \end{pmatrix} p^x q^{n-x}$$

We can use the expression we have just arrived at to compute the probability of x successes in the next n trials if we can find a way to calculate the number of ways to arrange x successes among n trials. It can be shown that:

The number of ways to arrange x *successes among* n *trials* equals

$$\frac{n!}{x!\,(n-x)!}$$

where $n!$ is pronounced "n factorial" and is calculated as $n! = n(n-1)(n-2)\cdots(1)$ and where (by definition) $0! = 1$.

For instance, using this formula, we can see that the number of ways to arrange $x = 2$ successes among $n = 3$ trials equals

$$\frac{n!}{x!\,(n-x)!} = \frac{3!}{2!\,(3-2)!} = \frac{3!}{2!\,1!} = \frac{3\cdot 2\cdot 1}{2\cdot 1\cdot 1} = 3$$

Of course, we have previously seen that the three ways to arrange $x = 2$ successes among $n = 3$ trials are *SSF*, *SFS*, and *FSS*.

Using the preceding formula, we obtain the following general result:

The Binomial Distribution

A **binomial experiment** has the following characteristics:

1. The experiment consists of *n identical trials.*
2. Each trial results in a **success** or a **failure.**
3. The probability of a success on any trial is p and remains constant from trial to trial. This implies that the probability of failure, q, on any trial is $1 - p$ and remains constant from trial to trial.
4. The trials are **independent** (that is, the results of the trials have nothing to do with each other).

Furthermore, if we define the random variable

x = the total number of successes in n trials of a binomial experiment

then we call x a **binomial random variable,** and the probability of obtaining x successes in n trials is

$$p(x) = \frac{n!}{x!\,(n-x)!}\,p^x q^{n-x}$$

Noting that we sometimes refer to the formula for $p(x)$ as the **binomial formula,** we illustrate the use of this formula in the following example.

EXAMPLE 5.7 Purchases at a Discount Store

Consider the discount department store situation discussed in Example 5.6. In order to find the probability that three of the next five customers will make purchases, we calculate

$$\begin{aligned} p(3) &= \frac{5!}{3!\,(5-3)!}(.4)^3(.6)^{5-3} = \frac{5!}{3!\,2!}(.4)^3(.6)^2 \\ &= \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(2\cdot 1)}(.4)^3(.6)^2 \\ &= 10(.064)(.36) \\ &= .2304 \end{aligned}$$

Here we see that

1 $\frac{5!}{3!\,(5-3)!} = 10$ is the number of ways to arrange three successes among five trials. For instance, two ways to do this are described by the sample space outcomes *SSSFF* and *SFSSF.* There are eight other ways.

2 $(.4)^3(.6)^2$ is the probability of any sample space outcome consisting of three successes and two failures.

Thus far we have shown how to calculate binomial probabilities. We next give several examples that illustrate some practical applications of the binomial distribution. As we demonstrate in the first example, the term *success* does not necessarily refer to a *desirable* experimental outcome. Rather, it refers to an outcome that we wish to investigate.

EXAMPLE 5.8 The Phe-Mycin Case: Drug Side Effects

Antibiotics occasionally cause nausea as a side effect. A major drug company has developed a new antibiotic called Phe-Mycin. The company claims that, at most, 10 percent of all patients treated with Phe-Mycin would experience nausea as a side effect of taking the drug. Suppose that we randomly select $n = 4$ patients and treat them with Phe-Mycin. Each patient will either experience nausea (which we arbitrarily call a success) or will not experience nausea (a failure). We will assume that p, the true probability that a patient will experience nausea as a side effect, is .10, the maximum value of p claimed by the drug company. Furthermore, it is reasonable to assume that patients' reactions to the drug would be independent of each other. Let x denote the number of patients among the four who will experience nausea as a side effect. It follows that x is a binomial random variable, which can take on any of the potential values 0, 1, 2, 3, or 4. That is, anywhere between none of the patients and all four of the patients could potentially experience nausea as a side effect. Furthermore, we can calculate the probability associated with each possible value of x as shown in Table 5.3. For instance, the probability that none of the four randomly selected patients will experience nausea is

$$\begin{aligned} p(0) = P(x = 0) &= \frac{4!}{0!\,(4-0)!}(.1)^0(.9)^{4-0} \\ &= \frac{4!}{0!\,4!}(.1)^0(.9)^4 \\ &= \frac{4!}{(1)(4!)}(1)(.9)^4 \\ &= (.9)^4 = .6561 \end{aligned}$$

Because Table 5.3 lists each possible value of x and also gives the probability of each value, we say that this table gives the **binomial probability distribution of x.**

TABLE 5.3 The Binomial Probability Distribution of x, the Number of Four Randomly Selected Patients Who Will Experience Nausea as a Side Effect of Being Treated with Phe-Mycin

x (Number Who Experience Nausea)	$p(x) = \frac{n!}{x!\,(n-x)!}p^x(1-p)^{n-x}$
0	$p(0) = P(x = 0) = \frac{4!}{0!\,(4-0)!}(.1)^0(.9)^{4-0} = .6561$
1	$p(1) = P(x = 1) = \frac{4!}{1!\,(4-1)!}(.1)^1(.9)^{4-1} = .2916$
2	$p(2) = P(x = 2) = \frac{4!}{2!\,(4-2)!}(.1)^2(.9)^{4-2} = .0486$
3	$p(3) = P(x = 3) = \frac{4!}{3!\,(4-3)!}(.1)^3(.9)^{4-3} = .0036$
4	$p(4) = P(x = 4) = \frac{4!}{4!\,(4-4)!}(.1)^4(.9)^{4-4} = .0001$

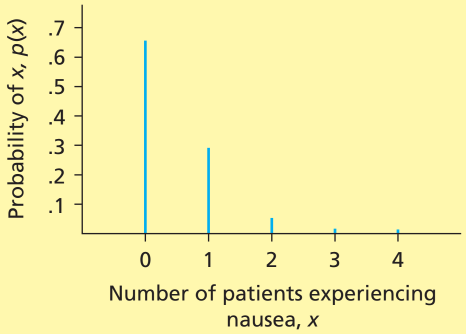
FIGURE 5.5 The Binomial Probability Distribution with $p = .10$ and $n = 4$

(a) Excel output of the binomial distribution

Binomial distribution with n = 4
and p = 0.10

x	P(X = x)
0	0.6561
1	0.2916
2	0.0486
3	0.0036
4	0.0001

(b) A graph of the distribution



The binomial probabilities given in Table 5.3 need not be hand calculated. Excel and MINITAB can be used to calculate binomial probabilities. For instance, Figure 5.5(a) gives the Excel output of the binomial probability distribution listed in Table 5.3.[1] Figure 5.5(b) shows a graph of this distribution.

In order to interpret these binomial probabilities, consider administering the antibiotic Phe-Mycin to all possible samples of four randomly selected patients. Then, for example,

$$P(x = 0) = 0.6561$$

says that none of the four sampled patients would experience nausea in 65.61 percent of all possible samples. Furthermore, as another example,

$$P(x = 3) = 0.0036$$

says that three out of the four sampled patients would experience nausea in only .36 percent of all possible samples.

Another way to avoid hand calculating binomial probabilities is to use **binomial tables,** which have been constructed to give the probability of x successes in n trials. A table of binomial

[1]As we will see in this chapter's appendixes, we can use MINITAB to obtain output of the binomial distribution that is essentially identical to the output given by Excel.

TABLE 5.4 A Portion of a Binomial Probability Table

(a) A Table for $n = 4$ Trials

Values of p (.05 to .50) ←

Number of Successes ↓	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	Number of Successes
0	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625	4
1	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500	3
2	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750	2
3	.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500	1
4	.0000	.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625	0
	.95	.90	.85	.80	.75	.70	.65	.60	.55	.50	↑

Values of p (.50 to .95) →

(b) A Table for $n = 8$ trials

Values of p (.05 to .50) ←

Number of Successes ↓	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	Number of Successes
0	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039	8
1	.2793	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0313	7
2	.0515	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094	6
3	.0054	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188	5
4	.0004	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734	4
5	.0000	.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188	3
6	.0000	.0000	.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094	2
7	.0000	.0000	.0000	.0001	.0004	.0012	.0033	.0079	.0164	.0313	1
8	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0007	.0017	.0039	0
	.95	.90	.85	.80	.75	.70	.65	.60	.55	.50	↑

Values of p (.50 to .95) →

probabilities is given in Table A.1 (page 783). A portion of this table is reproduced in Table 5.4(a) and (b). Part (a) of this table gives binomial probabilities corresponding to $n = 4$ trials. Values of p, the probability of success, are listed across the top of the table (ranging from $p = .05$ to $p = .50$ in steps of .05), and more values of p (ranging from $p = .50$ to $p = .95$ in steps of .05) are listed across the bottom of the table. When the value of p being considered is one of those across the top of the table, values of x (the number of successes in four trials) are listed down the left side of the table. For instance, to find the probabilities that we have computed in Table 5.3, we look in part (a) of Table 5.4 ($n = 4$) and read down the column labeled .10. Remembering that the values of x are on the left side of the table because $p = .10$ is on top of the table, we find the probabilities in Table 5.3 (they are shaded). For example, the probability that none of four patients will experience nausea is $p(0) = .6561$, the probability that one of the four patients will experience nausea is $p(1) = .2916$, and so forth. If the value of p is across the bottom of the table, then we read the values of x from the right side of the table. As an example, if p equals .70, then the probability of three successes in four trials is $p(3) = .4116$ (we have shaded this probability).

EXAMPLE 5.9 The Phe-Mycin Case: Drug Side Effects

Suppose that we wish to investigate whether p, the probability that a patient will experience nausea as a side effect of taking Phe-Mycin, is greater than .10, the maximum value of p claimed by the drug company. This assessment will be made by assuming, for the sake of argument, that p equals .10, and by using sample information to weigh the evidence against this assumption and in favor of the conclusion that p is greater than .10. Suppose that when a sample of $n = 4$ randomly selected patients is treated with Phe-Mycin, three of the four patients experience nausea. Because the fraction of patients in the sample that experience nausea is $3/4 = .75$, which is far greater than .10, we have some evidence contradicting the assumption that p equals .10. To evaluate the strength of this evidence, we calculate the probability that at least 3 out of 4 randomly selected patients would experience nausea as a side effect if, in fact, p equals .10. Using the binomial probabilities in Table 5.4(a), and realizing that the events $x = 3$ and $x = 4$ are mutually exclusive, we have

$$\begin{aligned} P(x \geq 3) &= P(x = 3 \text{ or } x = 4) \\ &= P(x = 3) + P(x = 4) \\ &= .0036 + .0001 \\ &= .0037 \end{aligned}$$

This probability says that, if p equals .10, then in only .37 percent of all possible samples of four randomly selected patients would at least three of the four patients experience nausea as a side effect. This implies that, if we are to believe that p equals .10, then we must believe that we have observed a sample result that is so rare that it can be described as a 37 in 10,000 chance. Because observing such a result is very unlikely, we have very strong evidence that p does not equal .10 and is, in fact, greater than .10.

Next, suppose that we consider what our conclusion would have been if only one of the four randomly selected patients had experienced nausea. Because the sample fraction of patients who experienced nausea is $1/4 = .25$, which is greater than .10, we would have some evidence to contradict the assumption that p equals .10. To evaluate the strength of this evidence, we calculate the probability that at least one out of four randomly selected patients would experience nausea as a side effect of being treated with Phe-Mycin if, in fact, p equals .10. Using the binomial probabilities in Table 5.4(a), we have

$$\begin{aligned} P(x \geq 1) &= P(x = 1 \text{ or } x = 2 \text{ or } x = 3 \text{ or } x = 4) \\ &= P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) \\ &= .2916 + .0486 + .0036 + .0001 \\ &= .3439 \end{aligned}$$

This probability says that, if p equals .10, then in 34.39 percent of all possible samples of four randomly selected patients, at least one of the four patients would experience nausea. Because it is not particularly difficult to believe that a 34.39 percent chance has occurred, we would not have much evidence against the claim that p equals .10.

Example 5.9 illustrates what is sometimes called the **rare event approach to making a statistical inference.** The idea of this approach is that if the probability of an observed sample result under a given assumption is *small,* then we have *strong evidence* that the assumption is false. Although there are no strict rules, many statisticians judge the probability of an observed sample result to be small if it is less than .05. The logic behind this will be explained more fully in Chapter 9.

EXAMPLE 5.10 The ColorSmart-5000 Case: TV Repairs

The manufacturer of the ColorSmart-5000 television set claims that 95 percent of its sets last at least five years without requiring a single repair. Suppose that we contact $n = 8$ randomly selected ColorSmart-5000 purchasers five years after they purchased their sets. Each purchaser's set will have needed no repairs (a success) or will have been repaired at least once (a failure). We will assume that p, the true probability that a purchaser's television set will require no repairs within five years, is .95, as claimed by the manufacturer. Furthermore, it is reasonable to believe that the repair records of the purchasers' sets are independent of each other. Let x denote the number of the $n = 8$ randomly selected sets that have lasted at least five years without a single repair. Then x is a binomial random variable that can take on any of the potential values 0, 1, 2, 3, 4, 5, 6, 7, or 8. The binomial distribution of x is listed in Table 5.5. Here we have obtained these probabilities from Table 5.4(b). To use Table 5.4(b), we look at the column corresponding to $p = .95$. Because $p = .95$ is listed at the bottom of the table, we read the values of x and their

TABLE 5.5 The Binomial Distribution of x, the Number of Eight ColorSmart-5000 Television Sets That Have Lasted at Least Five Years Without Needing a Single Repair, When $p = .95$

x, Number of Sets That Require No Repairs	$p(x) = \frac{8!}{x!\,(8-x)!}(.95)^x(.05)^{8-x}$
0	$p(0) = .0000$
1	$p(1) = .0000$
2	$p(2) = .0000$
3	$p(3) = .0000$
4	$p(4) = .0004$
5	$p(5) = .0054$
6	$p(6) = .0515$
7	$p(7) = .2793$
8	$p(8) = .6634$

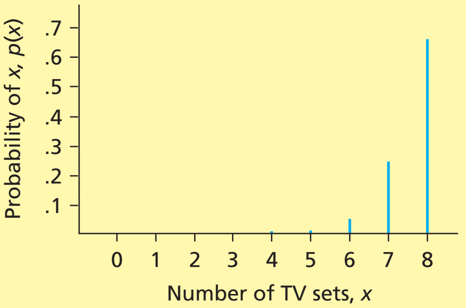
FIGURE 5.6 The Binomial Probability Distribution with $p = .95$ and $n = 8$

(a) MINITAB output of the binomial distribution

```
Binomial with n = 8 and p = 0.95

x        P( X = x )
3            0.0000
4            0.0004
5            0.0054
6            0.0515
7            0.2793
8            0.6634
```

(b) A graph of the distribution



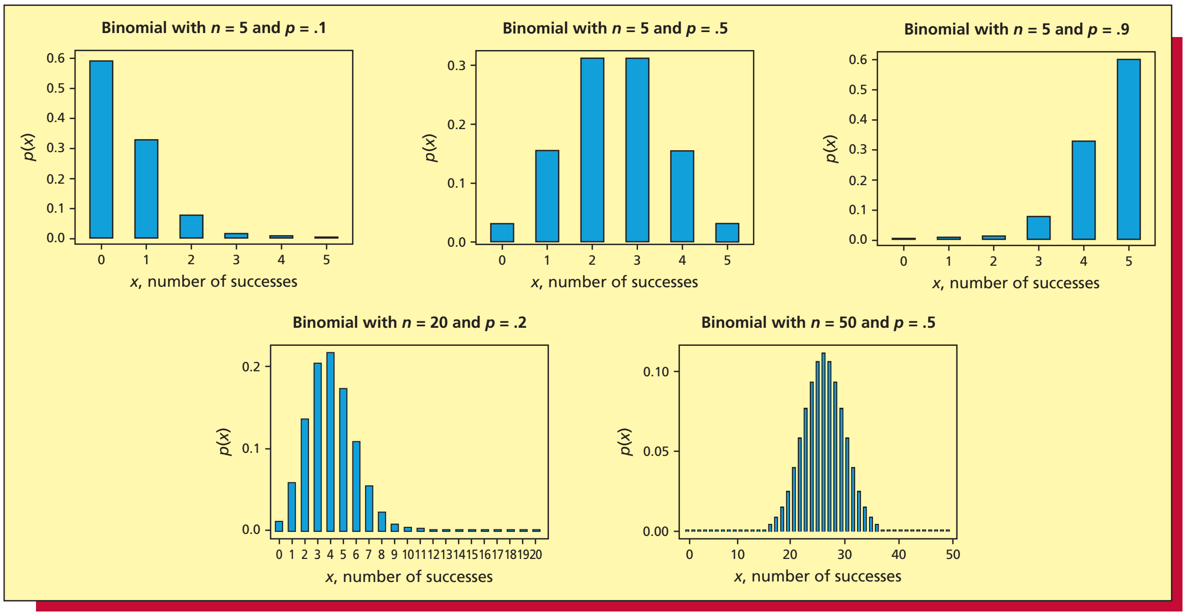
corresponding probabilities from bottom to top (we have shaded the probabilities). Notice that the values of x are listed on the right side of the table.

Figure 5.6(a) gives the MINITAB output of the binomial distribution with $p = .95$ and $n = 8$ (that is, the binomial distribution of Table 5.5). This binomial distribution is graphed in Figure 5.6(b). Now, suppose that when we actually contact eight randomly selected purchasers, we find that five out of the eight television sets owned by these purchasers have lasted at least five years without a single repair. Because the sample fraction, $5/8 = .625$, of television sets needing no repairs is less than .95, we have some evidence contradicting the manufacturer's claim that p equals .95. To evaluate the strength of this evidence, we will calculate the probability that five or fewer of the eight randomly selected televisions would last five years without a single repair if, in fact, p equals .95. Using the binomial probabilities in Table 5.5, we have

$$\begin{aligned}
P(x \le 5) &= P(x = 5 \text{ or } x = 4 \text{ or } x = 3 \text{ or } x = 2 \text{ or } x = 1 \text{ or } x = 0) \\
&= P(x = 5) + P(x = 4) + P(x = 3) + P(x = 2) + P(x = 1) + P(x = 0) \\
&= .0054 + .0004 + .0000 + .0000 + .0000 + .0000 \\
&= .0058
\end{aligned}$$

This probability says that, if p equals .95, then in only .58 percent of all possible samples of eight randomly selected ColorSmart-5000 televisions would five or fewer of the eight televisions last five years without a single repair. Therefore, if we are to believe that p equals .95, we must believe that a 58 in 10,000 chance has occurred. Because it is difficult to believe that such a small chance has occurred, we have strong evidence that p does not equal .95, and is, in fact, less than .95.

FIGURE 5.7 Several Binomial Distributions



In Examples 5.8 and 5.10 we have illustrated binomial distributions with different values of n and p. The values of n and p are often called the **parameters** of the binomial distribution. Figure 5.7 shows several different binomial distributions. We see that, depending on the parameters, a binomial distribution can be skewed to the right, skewed to the left, or symmetrical.

We next consider calculating the mean, variance, and standard deviation of a binomial random variable. If we place the binomial probability formula into the expressions (given in Section 5.2) for the mean and variance of a discrete random variable, we can derive formulas that allow us to easily compute μ_x, σ_x^2, and σ_x for a binomial random variable. Omitting the details of the derivation, we have the following results:

The Mean, Variance, and Standard Deviation of a Binomial Random Variable

If x is a binomial random variable, then

$$\mu_x = np \qquad \sigma_x^2 = npq \qquad \sigma_x = \sqrt{npq}$$

where n is the number of trials, p is the probability of success on each trial, and $q = 1 - p$ is the probability of failure on each trial.

As a simple example, again consider the television manufacturer, and recall that x is the number of eight randomly selected ColorSmart-5000 televisions that last five years without a single repair. If the manufacturer's claim that p equals .95 is true (which implies that q equals $1 - p = 1 - .95 = .05$), it follows that

$$\mu_x = np = 8(.95) = 7.6$$
$$\sigma_x^2 = npq = 8(.95)(.05) = .38$$
$$\sigma_x = \sqrt{npq} = \sqrt{.38} = .6164$$

In order to interpret $\mu_x = 7.6$, suppose that we were to randomly select all possible samples of eight ColorSmart-5000 televisions and record the number of sets in each sample that last five years without a repair. If we averaged all of our results, we would find that the average number of sets per sample that last five years without a repair is equal to 7.6.

To conclude this section, note that in optional Section 5.5, we discuss the **hypergeometric distribution.** This distribution is related to the binomial distribution. The main difference between the two distributions is that in the case of the hypergeometric distribution, the trials are not independent and the probabilities of success and failure change from trial to trial. This occurs when we sample without replacement from a finite population. However, when the finite population is large compared to the sample, the binomial distribution can be used to approximate the hypergeometric distribution. The details are explained in Section 5.5.

Exercises for Section 5.3

CONCEPTS

connect

5.18 List the four characteristics of a binomial experiment.

5.19 Suppose that x is a binomial random variable. Explain what the values of x represent. That is, how are the values of x defined?

5.20 Explain the logic behind the rare event approach to making statistical inferences.

METHODS AND APPLICATIONS

5.21 Suppose that x is a binomial random variable with $n = 5$, $p = .3$, and $q = .7$.

- **a** Write the binomial formula for this situation and list the possible values of x.
- **b** For each value of x, calculate $p(x)$, and graph the binomial distribution.
- **c** Find $P(x = 3)$.
- **d** Find $P(x \leq 3)$.
- **e** Find $P(x < 3)$.
- **f** Find $P(x \geq 4)$.
- **g** Find $P(x > 2)$.
- **h** Use the probabilities you computed in part *b* to calculate the mean, μ_x, the variance, σ_x^2, and the standard deviation, σ_x, of this binomial distribution. Show that the formulas for μ_x, σ_x^2, and σ_x given in this section give the same results.
- **i** Calculate the interval $[\mu_x \pm 2\sigma_x]$. Use the probabilities of part *b* to find the probability that x will be in this interval.

5.22 Thirty percent of all customers who enter a store will make a purchase. Suppose that six customers enter the store and that these customers make independent purchase decisions.

- **a** Let x = the number of the six customers who will make a purchase. Write the binomial formula for this situation.
- **b** Use the binomial formula to calculate
  - **(1)** The probability that exactly five customers make a purchase.
  - **(2)** The probability that at least three customers make a purchase.
  - **(3)** The probability that two or fewer customers make a purchase.
  - **(4)** The probability that at least one customer makes a purchase.

5.23 The customer service department for a wholesale electronics outlet claims that 90 percent of all customer complaints are resolved to the satisfaction of the customer. In order to test this claim, a random sample of 15 customers who have filed complaints is selected.

- **a** Let x = the number of sampled customers whose complaints were resolved to the customer's satisfaction. Assuming the claim is true, write the binomial formula for this situation.
- **b** Use the binomial tables (see Table A.1, page 783) to find each of the following if we assume that the claim is true:
  - **(1)** $P(x \leq 13)$.
  - **(2)** $P(x > 10)$.
  - **(3)** $P(x \geq 14)$.
  - **(4)** $P(9 \leq x \leq 12)$.
  - **(5)** $P(x \leq 9)$.
- **c** Suppose that of the 15 customers selected, 9 have had their complaints resolved satisfactorily. Using part *b*, do you believe the claim of 90 percent satisfaction? Explain.

5.24 The United States Golf Association requires that the weight of a golf ball must not exceed 1.62 oz. The association periodically checks golf balls sold in the United States by sampling specific brands stocked by pro shops. Suppose that a manufacturer claims that no more than 1 percent of its brand of golf balls exceed 1.62 oz. in weight. Suppose that 24 of this manufacturer's golf balls are randomly selected, and let x denote the number of the 24 randomly selected golf balls that exceed 1.62 oz. Figure 5.8 gives part of an Excel output of the binomial distribution with $n = 24$, $p = .01$, and $q = .99$. (Note that, because $P(X = x) = .0000$ for values of x from 6 to 24, we omit these probabilities.) Use this output to:

a Find $P(x = 0)$, that is, find the probability that none of the randomly selected golf balls exceeds 1.62 oz. in weight.

b Find the probability that at least one of the randomly selected golf balls exceeds 1.62 oz. in weight.

c Find $P(x \leq 3)$.

d Find $P(x \geq 2)$.

e Suppose that 2 of the 24 randomly selected golf balls are found to exceed 1.62 oz. Using your result from part *d*, do you believe the claim that no more than 1 percent of this brand of golf balls exceed 1.62 oz. in weight?

FIGURE 5.8 Excel Output of the Binomial Distribution with $n = 24$, $p = .01$, and $q = .99$

Binomial distribution with n = 24 and p = 0.01

x	P (X = x)
0	0.7857
1	0.1905
2	0.0221
3	0.0016
4	0.0001
5	0.0000

5.25 An industry representative claims that 50 percent of all satellite dish owners subscribe to at least one premium movie channel. In an attempt to justify this claim, the representative will poll a randomly selected sample of dish owners.

a Suppose that the representative's claim is true, and suppose that a sample of four dish owners is randomly selected. Assuming independence, use an appropriate formula to compute:

(1) The probability that none of the dish owners in the sample subscribes to at least one premium movie channel.

(2) The probability that more than two dish owners in the sample subscribe to at least one premium movie channel.

b Suppose that the representative's claim is true, and suppose that a sample of 20 dish owners is randomly selected. Assuming independence, what is the probability that:

(1) Nine or fewer dish owners in the sample subscribe to at least one premium movie channel?

(2) More than 11 dish owners in the sample subscribe to at least one premium movie channel?

(3) Fewer than five dish owners in the sample subscribe to at least one premium movie channel?

c Suppose that, when we survey 20 randomly selected dish owners, we find that 4 of the dish owners actually subscribe to at least one premium movie channel. Using a probability you found in this exercise as the basis for your answer, do you believe the industry representative's claim? Explain.

5.26 For each of the following, calculate μ_x, σ_x^2, and σ_x by using the formulas given in this section. Then (1) interpret the meaning of μ_x, and (2) find the probability that x falls in the interval $[\mu_x \pm 2\sigma_x]$.

a The situation of Exercise 5.22, where $x =$ the number of the six customers who will make a purchase.

b The situation of Exercise 5.23, where $x =$ the number of 15 sampled customers whose complaints were resolved to the customer's satisfaction.

c The situation of Exercise 5.24, where $x =$ the number of the 24 randomly selected golf balls that exceed 1.62 oz. in weight.

5.27 The January 1986 mission of the Space Shuttle *Challenger* was the 25th such shuttle mission. It was unsuccessful due to an explosion caused by an O-ring seal failure.

a According to NASA, the probability of such a failure in a single mission was 1/60,000. Using this value of p and assuming all missions are independent, calculate the probability of no mission failures in 25 attempts. Then calculate the probability of at least one mission failure in 25 attempts.

b According to a study conducted for the Air Force, the probability of such a failure in a single mission was 1/35. Recalculate the probability of no mission failures in 25 attempts and the probability of at least one mission failure in 25 attempts.

c Based on your answers to parts *a* and *b*, which value of p seems more likely to be true? Explain.

d How small must p be made in order to ensure that the probability of no mission failures in 25 attempts is .999?

5.4 The Poisson Distribution (Optional)

LO5-4 Use the Poisson distribution to compute probabilities (Optional).

We now discuss a discrete random variable that describes the number of occurrences of an event over a specified interval of time or space. For instance, we might wish to describe (1) the number of customers who arrive at the checkout counters of a grocery store in one hour, or (2) the number of major fires in a city during the next two months, or (3) the number of dirt specks found in one square yard of plastic wrap.

Such a random variable can often be described by a **Poisson distribution.** We describe this distribution and give two assumptions needed for its use in the following box:

The Poisson Distribution

Consider the number of times an event occurs over an interval of time or space, and assume that

1. The probability of the event's occurrence is the same for any two intervals of equal length, and
2. Whether the event occurs in any interval is independent of whether the event occurs in any other nonoverlapping interval.

Then, the probability that the event will occur x times in a *specified interval* is

$$p(x) = \frac{e^{-\mu}\mu^x}{x!}$$

Here μ is the mean (or expected) number of occurrences of the event in the *specified interval,* and $e = 2.71828\ldots$ is the base of Napierian logarithms.

In theory, there is no limit to how large x might be. That is, theoretically speaking, the event under consideration could occur an indefinitely large number of times during any specified interval. This says that a **Poisson random variable** might take on any of the values 0, 1, 2, 3, . . . and so forth. We will now look at an example.

EXAMPLE 5.11 The Air Safety Case: Traffic Control Errors

In an article in the August 15, 1998, edition of the *Journal News* (Hamilton, Ohio),[2] the Associated Press reported that the Cleveland Air Route Traffic Control Center, the busiest in the nation for guiding planes on cross-country routes, had experienced an unusually high number of errors since the end of July. An error occurs when controllers direct flights either within five miles of each other horizontally, or within 2,000 feet vertically at a height of 18,000 feet or more (the standard is 1,000 feet vertically at heights less than 18,000 feet). The controllers' union blamed the errors on a staff shortage, whereas the Federal Aviation Administration (FAA) claimed that the cause was improved error reporting and an unusual number of thunderstorms.

Suppose that an air traffic control center has been averaging 20.8 errors per year and that the center experiences 3 errors in a week. The FAA must decide whether this occurrence is unusual enough to warrant an investigation as to the causes of the (possible) increase in errors. To investigate this possibility, we will find the probability distribution of x, the number of errors in a week, when we assume that the center is still averaging 20.8 errors per year.

Arbitrarily choosing a time unit of one week, the average (or expected) number of errors per week is $20.8/52 = .4$. Therefore, we can use the Poisson formula (note that the Poisson assumptions are probably satisfied) to calculate the probability of no errors in a week to be

$$p(0) = P(x = 0) = \frac{e^{-\mu}\mu^0}{0!} = \frac{e^{-.4}(.4)^0}{1} = .6703$$

Similarly, the probability of three errors in a week is

$$p(3) = P(x = 3) = \frac{e^{-.4}(.4)^3}{3!} = \frac{e^{-.4}(.4)^3}{3 \cdot 2 \cdot 1} = .0072$$

As with the binomial distribution, tables have been constructed that give Poisson probabilities. A table of these probabilities is given in Table A.2 (page 787). A portion of this table is reproduced

[2] F. J. Frommer, "Errors on the Rise at Traffic Control Center in Ohio," *Journal News,* August 15, 1998.

TABLE 5.6 A Portion of a Poisson Probability Table

x, Number of Occurrences	μ, Mean Number of Occurrences .1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839
3	.0002	.0011	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613
4	.0000	.0001	.0003	.0007	.0016	.0030	.0050	.0077	.0111	.0153
5	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.0020	.0031
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005

x, Number of Occurrences	μ, Mean Number of Occurrences 1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	.3329	.3012	.2725	.2466	.2231	.2019	.1827	.1653	.1496	.1353
1	.3662	.3614	.3543	.3452	.3347	.3230	.3106	.2975	.2842	.2707
2	.2014	.2169	.2303	.2417	.2510	.2584	.2640	.2678	.2700	.2707
3	.0738	.0867	.0998	.1128	.1255	.1378	.1496	.1607	.1710	.1804
4	.0203	.0260	.0324	.0395	.0471	.0551	.0636	.0723	.0812	.0902
5	.0045	.0062	.0084	.0111	.0141	.0176	.0216	.0260	.0309	.0361
6	.0008	.0012	.0018	.0026	.0035	.0047	.0061	.0078	.0098	.0120
7	.0001	.0002	.0003	.0005	.0008	.0011	.0015	.0020	.0027	.0034
8	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0005	.0006	.0009

Source: From Brooks/Cole © 1991.

TABLE 5.7 The Poisson Distribution of x, the Number of Errors at an Air Traffic Control Center in a Week, When $\mu = .4$

x, the Number of Errors in a Week	$p(x) = \dfrac{e^{-\mu}\mu^x}{x!}$
0	$p(0) = \dfrac{e^{-.4}(.4)^0}{0!} = .6703$
1	$p(1) = \dfrac{e^{-.4}(.4)^1}{1!} = .2681$
2	$p(2) = \dfrac{e^{-.4}(.4)^2}{2!} = .0536$
3	$p(3) = \dfrac{e^{-.4}(.4)^3}{3!} = .0072$
4	$p(4) = \dfrac{e^{-.4}(.4)^4}{4!} = .0007$
5	$p(5) = \dfrac{e^{-.4}(.4)^5}{5!} = .0001$
6	$p(6) = \dfrac{e^{-.4}(.4)^6}{6!} = .0000$

in Table 5.6. In this table, values of the mean number of occurrences, μ, are listed across the top of the table, and values of x (the number of occurrences) are listed down the left side of the table. In order to use the table in the traffic control situation, we look at the column in Table 5.6 corresponding to .4, and we find the probabilities of 0, 1, 2, 3, 4, 5, and 6 errors (we have shaded these probabilities). For instance, the probability of one error in a week is .2681. Also, note that the probability of any number of errors greater than 6 is so small that it is not listed in the table. Table 5.7 summarizes the Poisson distribution of x, the number of errors in a week. This table also shows how the probabilities associated with the different values of x are calculated.

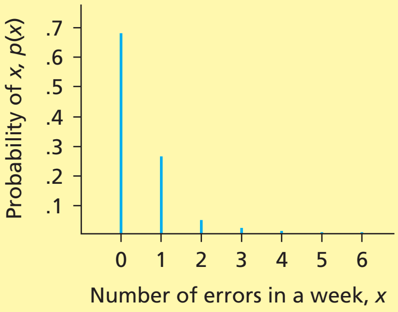
FIGURE 5.9 The Poisson Probability Distribution with $\mu = .4$

(a) MINITAB output of the Poisson distribution

```
Poisson with mean = 0.4

x      P( X = x )
0        0.6703
1        0.2681
2        0.0536
3        0.0072
4        0.0007
5        0.0001
6        0.0000
```

(b) A graph of the distribution



Poisson probabilities can also be calculated by using MINITAB and Excel. For instance, Figure 5.9(a) gives the MINITAB output of the Poisson distribution presented in Table 5.7.[3] This Poisson distribution is graphed in Figure 5.9(b).

Next, recall that there have been three errors at the air traffic control center in the last week. This is considerably more errors than .4, the expected number of errors assuming the center is still averaging 20.8 errors per year. Therefore, we have some evidence to contradict this assumption. To evaluate the strength of this evidence, we calculate the probability that at least three errors will occur in a week if, in fact, μ equals .4. Using the Poisson probabilities in Figure 5.9(a), we obtain

$$P(x \geq 3) = p(3) + p(4) + p(5) + p(6) = .0072 + .0007 + .0001 + .0000 = .008$$

This probability says that, if the center is averaging 20.8 errors per year, then there would be three or more errors in a week in only .8 percent of all weeks. That is, if we are to believe that the control center is averaging 20.8 errors per year, then we must believe that an 8 in 1,000 chance has occurred. Because it is very difficult to believe that such a rare event has occurred, we have strong evidence that the average number of errors per week has increased. Therefore, an investigation by the FAA into the reasons for such an increase is probably justified.

EXAMPLE 5.12 Errors in Computer Code

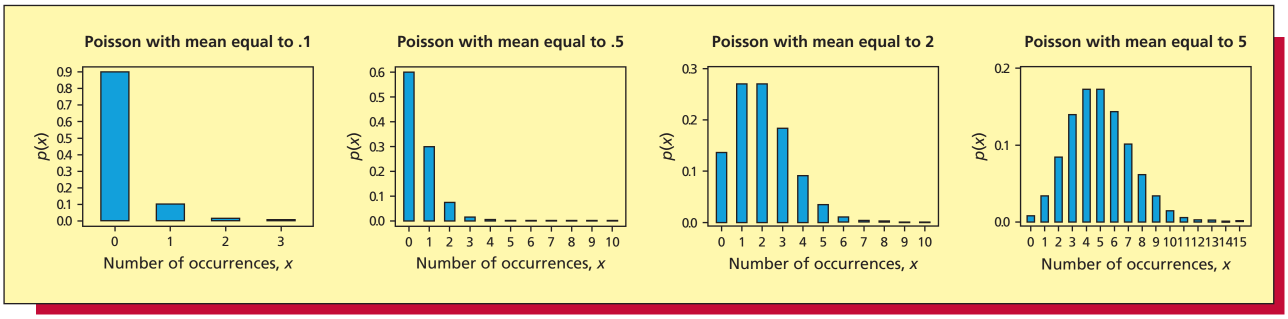
In the book *Modern Statistical Quality Control and Improvement,* Nicholas R. Farnum (1994) presents an example dealing with the quality of computer software. In the example, Farnum measures software quality by monitoring the number of errors per 1,000 lines of computer code.

Suppose that the number of errors per 1,000 lines of computer code is described by a Poisson distribution with a mean of four errors per 1,000 lines of code. If we wish to find the probability of obtaining eight errors in 2,500 lines of computer code, we must adjust the mean of the Poisson distribution. To do this, we arbitrarily choose a *space unit* of one line of code, and we note that a mean of four errors per 1,000 lines of code is equivalent to 4/1,000 of an error per line of code. Therefore, the mean number of errors per 2,500 lines of code is (4/1,000)(2,500) = 10. It follows that

$$p(8) = \frac{e^{-\mu}\mu^8}{8!} = \frac{e^{-10}10^8}{8!} = .1126$$

[3]As we will show in the appendixes to this chapter, we can use Excel and MegaStat to obtain output of the Poisson distribution that is essentially identical to the output given by MINITAB.

FIGURE 5.10 **Several Poisson Distributions**



The mean, μ, is often called the *parameter* of the Poisson distribution. Figure 5.10 shows several Poisson distributions. We see that, depending on its parameter (mean), a Poisson distribution can be very skewed to the right or can be quite symmetrical.

Finally, if we place the Poisson probability formula into the general expressions (of Section 5.2) for μ_x, σ_x^2, and σ_x, we can derive formulas for calculating the mean, variance, and standard deviation of a Poisson distribution:

The Mean, Variance, and Standard Deviation of a Poisson Random Variable

Suppose that x is a **Poisson random variable.** If μ is the average number of occurrences of an event over the specified interval of time or space of interest, then

$$\mu_x = \mu \qquad \sigma_x^2 = \mu \qquad \sigma_x = \sqrt{\mu}$$

Here we see that both the mean and the variance of a Poisson random variable equal the average number of occurrences μ of the event of interest over the specified interval of time or space. For example, in the air traffic control situation, the Poisson distribution of x, the number of errors at the air traffic control center in a week, has a mean of $\mu_x = .4$ and a standard deviation of $\sigma_x = \sqrt{.4} = .6325$.

Exercises for Section 5.4

CONCEPTS

connect

5.28 The values of a Poisson random variable are $x = 0, 1, 2, 3, \ldots$ Explain what these values represent.

5.29 Explain the assumptions that must be satisfied when a Poisson distribution adequately describes a random variable x.

METHODS AND APPLICATIONS

5.30 Suppose that x has a Poisson distribution with $\mu = 2$.
- **a** Write the Poisson formula and describe the possible values of x.
- **b** Starting with the smallest possible value of x, calculate $p(x)$ for each value of x until $p(x)$ becomes smaller than .001.
- **c** Graph the Poisson distribution using your results of b.
- **d** Find $P(x = 2)$.
- **e** Find $P(x \leq 4)$.
- **f** Find $P(x < 4)$.
- **g** Find $P(x \geq 1)$ and $P(x > 2)$.
- **h** Find $P(1 \leq x \leq 4)$.
- **i** Find $P(2 < x < 5)$.
- **j** Find $P(2 \leq x < 6)$.

5.31 Suppose that x has a Poisson distribution with $\mu = 2$.
- **a** Use the formulas given in this section to compute the mean, μ_x, variance, σ_x^2, and standard deviation, σ_x.
- **b** Calculate the intervals $[\mu_x \pm 2\sigma_x]$ and $[\mu_x \pm 3\sigma_x]$. Then use the probabilities you calculated in Exercise 5.30 to find the probability that x will be inside each of these intervals.

5.32 A bank manager wishes to provide prompt service for customers at the bank's drive-up window. The bank currently can serve up to 10 customers per 15-minute period without significant delay. The average arrival rate is 7 customers per 15-minute period. Let x denote the number of customers arriving per 15-minute period. Assuming x has a Poisson distribution:

a Find the probability that 10 customers will arrive in a particular 15-minute period.

b Find the probability that 10 or fewer customers will arrive in a particular 15-minute period.

c Find the probability that there will be a significant delay at the drive-up window. That is, find the probability that more than 10 customers will arrive during a particular 15-minute period.

5.33 A telephone company's goal is to have no more than five monthly line failures on any 100 miles of line. The company currently experiences an average of two monthly line failures per 50 miles of line. Let x denote the number of monthly line failures per 100 miles of line. Assuming x has a Poisson distribution:

a Find the probability that the company will meet its goal on a particular 100 miles of line.

b Find the probability that the company will not meet its goal on a particular 100 miles of line.

c Find the probability that the company will have no more than five monthly failures on a particular 200 miles of line.

d Find the probability that the company will have more than 12 monthly failures on a particular 150 miles of line.

5.34 A local law enforcement agency claims that the number of times that a patrol car passes through a particular neighborhood follows a Poisson process with a mean of three times per nightly shift. Let x denote the number of times that a patrol car passes through the neighborhood during a nightly shift.

a Calculate the probability that no patrol cars pass through the neighborhood during a nightly shift.

b Suppose that during a randomly selected night shift no patrol cars pass through the neighborhood. Based on your answer in part *a*, do you believe the agency's claim? Explain.

c Assuming that nightly shifts are independent and assuming that the agency's claim is correct, find the probability that exactly one patrol car will pass through the neighborhood on each of four consecutive nights.

5.35 When the number of trials, n, is large, binomial probability tables may not be available. Furthermore, if a computer is not available, hand calculations will be tedious. As an alternative, the Poisson distribution can be used to approximate the binomial distribution when n is large and p is small. Here the mean of the Poisson distribution is taken to be $\mu = np$. That is, when n is large and p is small, we can use the Poisson formula with $\mu = np$ to calculate binomial probabilities, and we will obtain results close to those we would obtain by using the binomial formula. A common rule is to use this approximation when $n/p \geq 500$.

To illustrate this approximation, in the movie *Coma,* a young female intern at a Boston hospital was very upset when her friend, a young nurse, went into a coma during routine anesthesia at the hospital. Upon investigation, she found that 10 of the last 30,000 healthy patients at the hospital had gone into comas during routine anesthesias. When she confronted the hospital administrator with this fact and the fact that the national average was 6 out of 100,000 healthy patients going into comas during routine anesthesias, the administrator replied that 10 out of 30,000 was still quite small and thus not that unusual.

a Use the Poisson distribution to approximate the probability that 10 or more of 30,000 healthy patients would slip into comas during routine anesthesias, if in fact the true average at the hospital was 6 in 100,000. Hint: $\mu = np = 30{,}000(6/100{,}000) = 1.8$.

b Given the hospital's record and part *a*, what conclusion would you draw about the hospital's medical practices regarding anesthesia?

(Note: It turned out that the hospital administrator was part of a conspiracy to sell body parts and was purposely putting healthy adults into comas during routine anesthesias. If the intern had taken a statistics course, she could have avoided a great deal of danger.)

5.36 Suppose that an automobile parts wholesaler claims that .5 percent of the car batteries in a shipment are defective. A random sample of 200 batteries is taken, and four are found to be defective.

a Use the Poisson approximation discussed in Exercise 5.35 to find the probability that four or more car batteries in a random sample of 200 such batteries would be found to be defective, if we assume that the wholesaler's claim is true.

b Based on your answer to part *a*, do you believe the claim? Explain.

5.5 The Hypergeometric Distribution (Optional)

The Hypergeometric Distribution

Suppose that a population consists of N items and that r of these items are *successes* and $(N - r)$ of these items are *failures*. If we randomly select n of the N items **without replacement**, it can be shown that the probability that x of the n randomly selected items will be successes is given by the **hypergeometric probability formula**

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

LO5-5 Use the hypergeometric distribution to compute probabilities (Optional).

Here $\binom{r}{x}$ is the number of ways x successes can be selected from the total of r successes in the population, $\binom{N-r}{n-x}$ is the number of ways $n - x$ failures can be selected from the total of $N - r$ failures in the pupulation, and $\binom{N}{n}$ is the number of ways a sample of size n can be selected from a population of size N.

To demonstrate the calculations, suppose that a population of $N = 6$ stocks consists of $r = 4$ stocks that are destined to give positive returns (that is, there are $r = 4$ *successes*) and $N - r = 6 - 4 = 2$ stocks that are destined to give negative returns (that is, there are $N - r = 2$ *failures*). Also suppose that we randomly select $n = 3$ of the six stocks in the population without replacement and that we define x to be the number of the three randomly selected stocks that will give a positive return. Then, for example, the probability that $x = 2$ is

$$p(x = 2) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}} = \frac{\left(\frac{4!}{2!\,2!}\right)\left(\frac{2!}{1!\,1!}\right)}{\left(\frac{6!}{3!\,3!}\right)} = \frac{(6)(2)}{20} = .6$$

Similarly, the probability that $x = 3$ is

$$p(x = 3) = \frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}} = \frac{\left(\frac{4!}{3!\,1!}\right)\left(\frac{2!}{0!\,2!}\right)}{\left(\frac{6!}{3!\,3!}\right)} = \frac{(4)(1)}{20} = .2$$

It follows that the probability that at least two of the three randomly selected stocks will give a positive return is $p(x = 2) + p(x = 3) = .6 + .2 = .8$.

If we place the hypergeometric probability formula into the general expressions (of Section 5.2) for μ_x and σ_x^2, we can derive formulas for the mean and variance of the hypergeometric distribution.

The Mean and Variance of a Hypergeometric Random Variable

Suppose that x is a hypergeometric random variable. Then

$$\mu_x = n\left(\frac{r}{N}\right) \quad \text{and} \quad \sigma_x^2 = n\left(\frac{r}{N}\right)\left(1 - \frac{r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

In the previous example, we have $N = 6$, $r = 4$, and $n = 3$. It follows that

$$\mu_x = n\left(\frac{r}{N}\right) = 3\left(\frac{4}{6}\right) = 2, \quad \text{and}$$

$$\sigma_x^2 = n\left(\frac{r}{N}\right)\left(1 - \frac{r}{N}\right)\left(\frac{N-n}{N-1}\right) = 3\left(\frac{4}{6}\right)\left(1 - \frac{4}{6}\right)\left(\frac{6-3}{6-1}\right) = .4$$

and that the standard deviation $\sigma_x = \sqrt{.4} = .6325$.

To conclude this section, note that, on the first random selection from the population of N items, the probability of a success is r/N. Because we are making selections *without replacement,* the probability of a success changes as we continue to make selections. However, **if the population size N is "much larger" than the sample size n (say, at least 20 times as large),** then making the selections will not substantially change the probability of a success. In this case, we can assume that the probability of a success stays essentially constant from selection to selection, and the different selections are essentially independent of each other. Therefore, **we can approximate the hypergeometric distribution by the binomial distribution.** That is, we can compute probabilities about the hypergeometric random variable x by using the easier binomial probability formula

$$p(x) = \frac{n!}{x!\,(n-x)!}p^x(1-p)^{n-x} = \frac{n!}{x!\,(n-x)!}\left(\frac{r}{N}\right)^x\left(1-\frac{r}{N}\right)^{n-x}$$

where the binomial probability of success equals r/N.

EXAMPLE 5.13 Big Screen TV Repairs

Suppose that you own an electronics store and purchase (randomly select) 15 big screen TVs from a production run of 500. Of the 500 TVs, 450 are destined to last at least five years without needing a single repair. We will find the *exact* probability that at least 14 of the 15 big screen TVs will last at least five years without needing a single repair. If we let x = the number of TVs that will last at least five years without needing a single repair, then x follows a hypergeometric distribution and $P(x \geq 14)$ is given by

$$P(x \geq 14) = P(x = 14) + P(x = 15) = p(14) + p(15).$$

Placing the values of $N = 500$, $r = 450$, $N - r = 50$, and $n = 15$ into the hypergeometric formula, and setting x equal to 14, we get

$$p(14) = \frac{\binom{450}{14}\binom{50}{1}}{\binom{500}{15}} = \frac{\left(\frac{450!}{14!\,436!}\right)\left(\frac{50!}{1!\,49!}\right)}{\frac{500!}{15!\,485!}}$$

Then, setting $x = 15$, we get

$$p(15) = \frac{\binom{450}{15}\binom{50}{0}}{\binom{500}{15}} = \frac{\left(\frac{450!}{15!\,435!}\right)\left(\frac{50!}{0!\,50!}\right)}{\frac{500!}{15!\,485!}}$$

A standard calculator may not be able to handle these calculations. Using a computer, we get $P(x \geq 14) = p(14) + p(15) = 0.5469$.

As can be seen in this example, the calculation of hypergeometric probabilities can be difficult. However, because the population size $N = 500$ is at least 20 times the sample size $n = 15$, we can approximate this probability by using the binomial approximation to the hypergeometric distribution. Here, $p = 450/500 = 0.9$. Using this approximation, we get $P(x \geq 14) = p(14) + p(15)$, where

$$p(x) = \frac{15!}{x!\,(15-x)!}(0.9^x)(0.1^{15-x})$$

Using Table A.1 on page 783, where $n = 15$ and $p = .9$, we get $p(14) = .3432$ and $p(15) = .2059$. This gives us $P(x \geq 14) = .3432 + .2059 = .5491$. When we use the binomial approximation to find a hypergeometric probability, the calculations (either by formula or table) are much easier. Further, if used appropriately, the binomial approximation will give a result that is very close to the exact (hypergeometric) result. In this example, the results are the same to two decimal places.

Exercises for Section 5.5

CONCEPTS

5.37 In the context of the hypergeometric distribution, explain the meanings of N, r, and n.

5.38 When can a hypergeometric distribution be approximated by a binomial distribution? Explain carefully what this means.

METHODS AND APPLICATIONS

5.39 Suppose that x has a hypergeometric distribution with $N = 8$, $r = 5$, and $n = 4$. Find:
- **a** $P(x = 0)$
- **b** $P(x = 1)$
- **c** $P(x = 2)$
- **d** $P(x = 3)$
- **e** $P(x = 4)$
- **f** $P(x \geq 2)$
- **g** $P(x < 3)$
- **h** $P(x > 1)$

5.40 Suppose that x has a hypergeometric distribution with $N = 10$, $r = 4$, and $n = 3$.
- **a** Write out the probability distribution of x.
- **b** Find the mean μ_x, variance σ_x^2, and standard deviation σ_x of this distribution.

5.41 Among 12 metal parts produced in a machine shop, 3 are defective. If a random sample of three of these metal parts is selected, find:
- **a** The probability that this sample will contain at least two defectives.
- **b** The probability that this sample will contain at most one defective.

5.42 Suppose that you purchase (randomly select) 3 TV sets from a production run of 10 TV sets. Of the 10 TV sets, 9 are destined to last at least five years without needing a single repair. What is the probability that all three of your TV sets will last at least five years without needing a single repair?

5.43 Suppose that you own a car dealership and purchase (randomly select) 7 cars of a certain make from a production run of 200 cars. Of the 200 cars, 160 are destined to last at least five years without needing a major repair. Set up an expression using the hypergeometric distribution for the probability that at least 6 of your 7 cars will last at least five years without needing a major repair. Then, using the binomial tables (see Table A.1, page 783), approximate this probability by using the binomial distribution. What justifies the approximation? Hint: $p = r/N = 160/200 = .8$.

5.6 Joint Distributions and the Covariance (Optional)

LO5-6 Compute and understand the covariance between two random variables (Optional).

Below we present (1) the probability distribution of x, the yearly proportional return for stock A, (2) the probability distribution of y, the yearly proportional return for stock B, and (3) the **joint probability distribution of (x, y),** the joint yearly proportional returns for stocks A and B [note that we have obtained the data below from Pfaffenberger and Patterson (1987)].

x	$p(x)$
−0.10	0.400
0.05	0.125
0.15	0.100
0.38	0.375

$\mu_x = .124$, $\sigma_x^2 = .0454$, $\sigma_x = .2131$

y	$p(y)$
−0.15	0.300
−0.05	0.200
0.12	0.150
0.46	0.350

$\mu_y = .124$, $\sigma_y^2 = .0681$, $\sigma_y = .2610$

Joint Distribution of (x, y)

Stock B Return, y	Stock A Return, x: −0.10	0.05	0.15	0.38
−0.15	0.025	0.025	0.025	0.225
−0.05	0.075	0.025	0.025	0.075
0.12	0.050	0.025	0.025	0.050
0.46	$\boxed{0.250}$	0.050	0.025	0.025

To explain the joint probability distribution, note that the probability of .250 enclosed in the rectangle is the probability that in a given year the return for stock A will be −.10 and the return for stock B will be .46. The probability of .225 enclosed in the oval is the probability that in a given year the return for stock A will be .38 and the return for stock B will be −.15. Intuitively, these two rather large probabilities say that (1) a negative return x for stock A tends to be associated with a highly positive return y for stock B, and (2) a highly positive return x for stock A tends to be associated with a negative return y for stock B. To further measure the association between x and y, we can calculate the *covariance* between x and y. To do this, we calculate $(x - \mu_x)(y - \mu_y) = (x - .124)(y - .124)$ for each combination of values of x and y. Then, we multiply each $(x - \mu_x)(y - \mu_y)$ value by the probability $p(x, y)$ of the (x, y) combination of values and add up the quantities that we obtain. The resulting number is the **covariance,** denoted $\boldsymbol{\sigma^2_{xy}}$. For example, for the combination of values $x = -.10$ and $y = .46$, we calculate

$$(x - \mu_x)(y - \mu_y)\, p(x, y) = (-.10 - .124)(.46 - .124)(.250) = -.0188$$

Doing this for all combinations of (x, y) values and adding up the resulting quantities, we find that the covariance is $-.0318$. In general, a negative covariance says that as x increases, y tends to decrease in a linear fashion. A positive covariance says that as x increases, y tends to increase in a linear fashion.

In this situation, the covariance helps us to understand the importance of investment diversification. If we invest all of our money in stock A, we have seen that $\mu_x = .124$ and $\sigma_x = .2131$. If we invest all of our money in stock B, we have seen that $\mu_y = .124$ and $\sigma_y = .2610$. If we invest half of our money in stock A and half of our money in stock B, the return for the portfolio is $P = .5x + .5y$. To find the expected value of the portfolio return, we need to use a *property of expected values*. This property says if a and b are constants, and if x and y are random variables, then

$$\mu_{(ax+by)} = a\mu_x + b\mu_y$$

Therefore,

$$\mu_P = \mu_{(.5x+.5y)} = .5\mu_x + .5\mu_y = .5(.124) + .5(.124) = .124$$

To find the variance of the portfolio return, we must use a *property of variances*. In general, if x and y have a covariance σ^2_{xy}, and a and b are constants, then

$$\sigma^2_{(ax+by)} = a^2\sigma^2_x + b^2\sigma^2_y + 2ab\sigma^2_{xy}$$

Therefore

$$\sigma^2_P = \sigma^2_{(.5x+.5y)} = (.5)^2\sigma^2_x + (.5)^2\sigma^2_y + 2(.5)(.5)\sigma^2_{xy}$$

$$= (.5)^2(.0454) + (.5)^2(.0681) + 2(.5)(.5)(-.0318) = .012475$$

and $\sigma_P = \sqrt{.012475} = .1117$. Note that, because $\mu_P = .124$ equals $\mu_x = .124$ and $\mu_y = .124$, the portfolio has the same expected return as either stock A or B. However, because $\sigma_P = .1117$ is less than $\sigma_x = .2131$ and $\sigma_y = .2610$, the portfolio is a less risky investment. In other words, diversification *can* reduce risk. Note, however, that the reason that σ_P is *less* than σ_x and σ_y is that $\sigma^2_{xy} = -.0318$ *is negative*. Intuitively, this says that the two stocks tend to balance each other's returns. However, if the covariance between the returns of two stocks is positive, σ_P can be larger than σ_x and/or σ_y. The student will demonstrate this in Exercise 5.46.

Next, note that a measure of linear association between x and y that is unitless and always between -1 and 1 is the **correlation coefficient,** denoted ρ. We define ρ as follows:

The **correlation coefficient** between x and y is $\rho = \sigma^2_{xy}/\sigma_x\sigma_y$

For the stock return example, ρ equals $(-.0318)/((.2131)(.2610)) = -.5717$.

To conclude this section, we summarize four properties of expected values and variances that we will use in optional Section 7.6 to derive some important facts about the sample mean:

Property 1: If a is a constant and x is a random variable, $\mu_{ax} = a\mu_x$.

Property 2: If $x_1, x_2, \ldots, x_n$ are random variables, $\mu_{(x_1+x_2+\cdots+x_n)} = \mu_{x_1} + \mu_{x_2} + \cdots + \mu_{x_n}$.

Property 3: If a is a constant and x is a random variable, $\sigma^2_{ax} = a^2\sigma^2_x$.

Property 4: If $x_1, x_2, \ldots, x_n$ are statistically independent random variables (that is, if the value taken by any one of these independent variables is in no way associated with the value taken by any other of these random variables), then the covariance between any two of these random variables is zero and $\sigma^2_{(x_1+x_2+\cdots+x_n)} = \sigma^2_{x_1} + \sigma^2_{x_2} + \cdots + \sigma^2_{x_n}$.

Exercises for Section 5.6

CONCEPTS

5.44 Explain the meaning of a negative covariance.

5.45 Explain the meaning of a positive covariance.

METHODS AND APPLICATIONS

5.46 Let x be the yearly proportional return for stock C, and let y be the yearly proportional return for stock D. If $\mu_x = .11$, $\mu_y = .09$, $\sigma_x = .17$, $\sigma_y = .17$, and $\sigma^2_{xy} = .0412$, find the mean and standard deviation of the portfolio return $P = .5x + .5y$. Discuss the risk of the portfolio.

5.47 Below we give a joint probability table for two utility bonds where the random variable x represents the percentage return for bond 1 and the random variable y represents the percentage return for bond 2.

			x			
y	**8**	**9**	**10**	**11**	**12**	**p(y)**
8	.03	.04	.03	.00	.00	.10
9	.04	.06	.06	.04	.00	.20
10	.02	.08	.20	.08	.02	.40
11	.00	.04	.06	.06	.04	.20
12	.00	.00	.03	.04	.03	.10
p(x)	.09	.22	.38	.22	.09	

Source: David K. Hildebrand and Lyman Ott, *Statistical Thinking for Managers,* 2nd edition (Boston, MA: Duxbury Press, 1987), p. 101.

In this table, probabilities associated with values of x are given in the row labeled $p(x)$ and probabilities associated with values of y are given in the column labeled $p(y)$. For example, $P(x = 9) = .22$ and $P(y = 11) = .20$. The entries inside the body of the table are joint probabilities—for instance, the probability that x equals 9 and y equals 10 is .08. Use the table to do the following:

a Calculate μ_x, σ_x, μ_y, σ_y, and σ^2_{xy}.

b Calculate the variance and standard deviation of a portfolio in which 50 percent of the money is used to buy bond 1 and 50 percent is used to buy bond 2. That is, find σ^2_P and σ_P, where $P = .5x + .5y$. Discuss the risk of the portfolio.

Chapter Summary

In this chapter we began our study of **random variables.** We learned that **a random variable represents an uncertain numerical outcome.** We also learned that a random variable whose values can be listed is called a **discrete random variable,** while the values of a **continuous random variable** correspond to one or more intervals on the real number line. We saw that a **probability distribution** of a discrete random variable is a table, graph, or formula that gives the probability associated with each of the random variable's possible values. We also discussed several descriptive measures of a discrete random variable—its **mean** (or **expected value**), its **variance,** and its s**tandard deviation.** We continued this chapter by studying two important, commonly used discrete probability distributions—the **binomial distribution** and the **Poisson distribution**—and we demonstrated how these distributions can be used to make statistical inferences. Finally, we studied a third important discrete probability distribution, the **hypergeometric distribution,** and we discussed **joint distributions** and the **covariance.**

Glossary of Terms

binomial distribution: The probability distribution that describes a binomial random variable. (page 199)

binomial experiment: An experiment that consists of n independent, identical trials, each of which results in either a success or a failure and is such that the probability of success on any trial is the same. (page 199)

binomial random variable: A random variable that is defined to be the total number of successes in n trials of a binomial experiment. (page 199)

binomial tables: Tables in which we can look up binomial probabilities. (pages 201, 202)

continuous random variable: A random variable whose values correspond to one or more intervals of numbers on the real number line. (page 187)

correlation coefficient: A unitless measure of the linear relationship between two random variables. (page 216)

covariance: A non-unitless measure of the linear relationship between two random variables. (page 216)

discrete random variable: A random variable whose values can be counted or listed. (page 187)
expected value (of a random variable): The mean of the population of all possible observed values of a random variable. That is, the long-run average value obtained if values of a random variable are observed a (theoretically) infinite number of times. (page 190)
hypergeometric distribution: The probability distribution that describes a hypergeometric random variable. (page 213)
hypergeometric random variable: A random variable that is defined to be the number of successes obtained in a random sample selected without replacement from a finite population of N elements that contains r successes and $N - r$ failures. (page 213)
joint probability distribution of (x, y): A probability distribution that assigns probabilities to all combinations of values of x and y. (page 215)
Poisson distribution: The probability distribution that describes a Poisson random variable. (page 208)
Poisson random variable: A discrete random variable that can often be used to describe the number of occurrences of an event over a specified interval of time or space. (page 208)
probability distribution (of a discrete random variable): A table, graph, or formula that gives the probability associated with each of the random variable's values. (page 188)
random variable: A variable that assumes numerical values that are determined by the outcome of an experiment. That is, a variable that represents an uncertain numerical outcome. (page 187)
standard deviation (of a random variable): The standard deviation of the population of all possible observed values of a random variable. It measures the spread of the population of all possible observed values of the random variable. (page 192)
variance (of a random variable): The variance of the population of all possible observed values of a random variable. It measures the spread of the population of all possible observed values of the random variable. (page 192)

Important Formulas

Properties of a discrete probability distribution: page 189
The mean (expected value) of a discrete random variable: page 190
Variance and standard deviation of a discrete random variable: page 192
Binomial probability formula: page 199
Mean, variance, and standard deviation of a binomial random variable: page 205
Poisson probability formula: page 208
Mean, variance, and standard deviation of a Poisson random variable: page 211
Hypergeometric probability formula: page 213
Mean and variance of a hypergeometric random variable: page 213
Correlation coefficient between x and y: page 216
Properties of expected values and variances: page 216

Supplementary Exercises

5.48 A rock concert promoter has scheduled an outdoor concert on July 4th. If it does not rain, the promoter will make $30,000. If it does rain, the promoter will lose $15,000 in guarantees made to the band and other expenses. The probability of rain on the 4th is .4.
- **a** What is the promoter's expected profit? Is the expected profit a reasonable decision criterion? Explain.
- **b** How much should an insurance company charge to insure the promoter's full losses? Explain your answer.

5.49 The demand (in number of copies per day) for a city newspaper, x, has historically been 50,000, 70,000, 90,000, 110,000, or 130,000 with the respective probabilities .1, .25, .4, .2, and .05.
- **a** Graph the probability distribution of x.
- **b** Find the expected demand. Interpret this value, and label it on the graph of part *a*.
- **c** Using Chebyshev's Theorem, find the minimum percentage of all possible daily demand values that will fall in the interval $[\mu_x \pm 2\sigma_x]$.
- **d** Calculate the interval $[\mu_x \pm 2\sigma_x]$. Illustrate this interval on the graph of part *a*. According to the probability distribution of demand x previously given, what percentage of all possible daily demand values fall in the interval $[\mu_x \pm 2\sigma_x]$?

5.50 United Medicine, Inc., claims that a drug, Viro, significantly relieves the symptoms of a certain viral infection for 80 percent of all patients. Suppose that this drug is given to eight randomly selected patients who have been diagnosed with the viral infection.
- **a** Let x equal the number of the eight randomly selected patients whose symptoms are significantly relieved. What distribution describes the random variable x? Explain.
- **b** Assuming that the company's claim is correct, find $P(x \leq 3)$.
- **c** Suppose that of the eight randomly selected patients, three have had their symptoms significantly relieved by Viro. Based on the probability in part *b*, would you believe the claim of United Medicine, Inc.? Explain.

5.51 A consumer advocate claims that 80 percent of cable television subscribers are not satisfied with their cable service. In an attempt to justify this claim, a randomly selected sample of cable subscribers will be polled on this issue.

a Suppose that the advocate's claim is true, and suppose that a random sample of five cable subscribers is selected. Assuming independence, use an appropriate formula to compute the probability that four or more subscribers in the sample are not satisfied with their service.

b Suppose that the advocate's claim is true, and suppose that a random sample of 25 cable subscribers is selected. Assuming independence, find:

(1) The probability that 15 or fewer subscribers in the sample are not satisfied with their service.

(2) The probability that more than 20 subscribers in the sample are not satisfied with their service.

(3) The probability that between 20 and 24 (inclusive) subscribers in the sample are not satisfied with their service.

(4) The probability that exactly 24 subscribers in the sample are not satisfied with their service.

c Suppose that when we survey 25 randomly selected cable television subscribers, we find that 15 are actually not satisfied with their service. Using a probability you found in this exercise as the basis for your answer, do you believe the consumer advocate's claim? Explain.

5.52 A retail store has implemented procedures aimed at reducing the number of bad checks cashed by its cashiers. The store's goal is to cash no more than eight bad checks per week. The average number of bad checks cashed is three per week. Let x denote the number of bad checks cashed per week. Assuming that x has a Poisson distribution:

a Find the probability that the store's cashiers will not cash any bad checks in a particular week.

b Find the probability that the store will meet its goal during a particular week.

c Find the probability that the store will not meet its goal during a particular week.

d Find the probability that the store's cashiers will cash no more than 10 bad checks per two-week period.

e Find the probability that the store's cashiers will cash no more than five bad checks per three-week period.

5.53 Suppose that the number of accidents occurring in an industrial plant is described by a Poisson process with an average of 1.5 accidents every three months. During the last three months, four accidents occurred.

a Find the probability that no accidents will occur during the current three-month period.

b Find the probability that fewer accidents will occur during the current three-month period than occurred during the last three-month period.

c Find the probability that no more than 12 accidents will occur during a particular year.

d Find the probability that no accidents will occur during a particular year.

5.54 A high-security government installation has installed four security systems to detect attempted break-ins. The four security systems operate independently of each other, and each has a .85 probability of detecting an attempted break-in. Assume an attempted break-in occurs. Use the binomial distribution to find the probability that at least one of the four security systems will detect it.

5.55 A new stain removal product claims to completely remove the stains on 90 percent of all stained garments. Assume that the product will be tested on 20 randomly selected stained garments, and let x denote the number of these garments from which the stains will be completely removed. Use the binomial distribution to find $P(x \leq 13)$ if the stain removal product's claim is correct. If x actually turns out to be 13, what do you think of the claim?

5.56 Consider Exercise 5.55, and find $P(x \leq 17)$ if the stain removal product's claim is correct. If x actually turns out to be 17, what do you think of the claim?

5.57 A state has averaged one small business failure per week over the past several years. Let x denote the number of small business failures in the next eight weeks. Use the Poisson distribution to find $P(x \geq 17)$ if the mean number of small business failures remains what it has been. If x actually turns out to be 17, what does this imply?

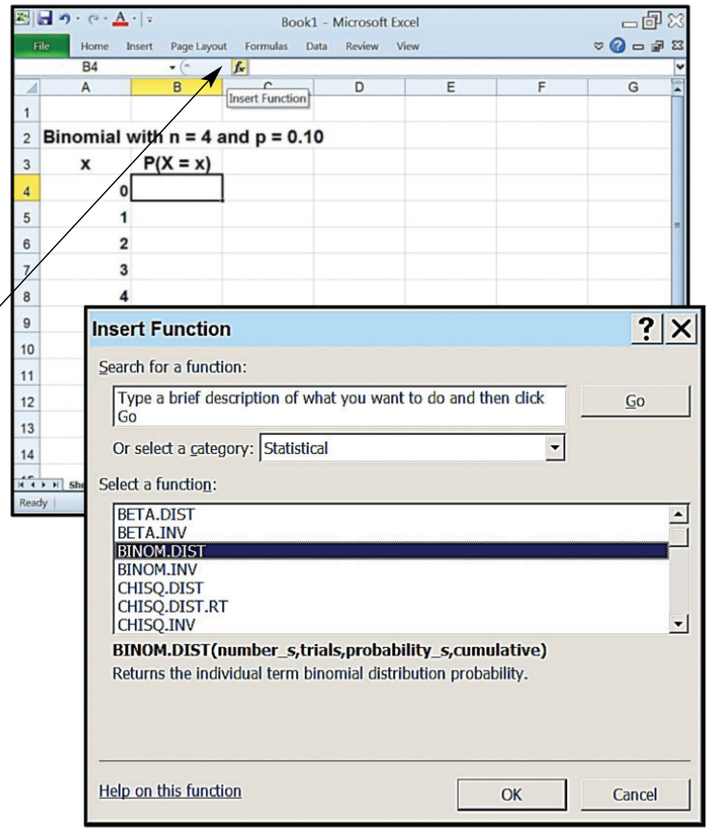
5.58 A candy company claims that its new chocolate almond bar averages 10 almonds per bar. Let x denote the number of almonds in the next bar that you buy. Use the Poisson distribution to find $P(x \leq 4)$ if the candy company's claim is correct. If x actually turns out to be 4, what do you think of the claim?

5.59 Consider Exercise 5.58, and find $P(x \leq 8)$ if the candy company's claim is true. If x actually turns out to be 8, what do you think of the claim?

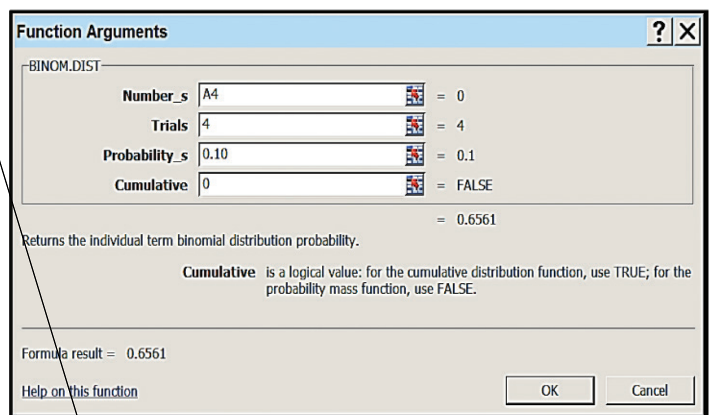
Appendix 5.1 ■ Binomial, Poisson, and Hypergeometric Probabilities Using Excel

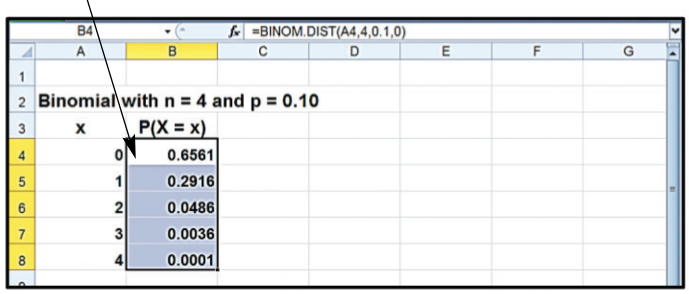
Binomial probabilities in Figure 5.5(a) on page 201:

- Enter the title, "Binomial with n = 4 and p = 0.10," in the cell location where you wish to place the binomial results. We have placed the title beginning in cell A2 (any other choice will do).
- In cell A3, enter the heading, x.
- Enter the values 0 through 4 in cells A4 through A8.
- In cell B3, enter the heading P(X = x).
- Click in cell B4 (this is where the first binomial probability will be placed). Click on the Insert Function button $\boxed{f_x}$ on the Excel toolbar.
- In the Insert Function dialog box, select Statistical from the "Or select a category:" menu, select BINOM.DIST from the "Select a function:" menu, and click OK.



- In the BINOM.DIST Function Arguments dialog box, enter the cell location A4 (this cell contains the value for which the first binomial probability will be calculated) in the "Number_s" window.
- Enter the value 4 in the Trials window.
- Enter the value 0.10 in the "Probability_s" window.
- Enter the value 0 in the Cumulative window.
- Click OK in the BINOM.DIST Function Arguments dialog box.
- When you click OK, the calculated result (0.6561) will appear in cell B4. Double-click the drag handle (in the lower right corner) of cell B4 to automatically extend the cell formula to cells B5 through B8.
- The remaining probabilities will be placed in cells B5 through B8.





To obtain **hypergeometric probabilities:**

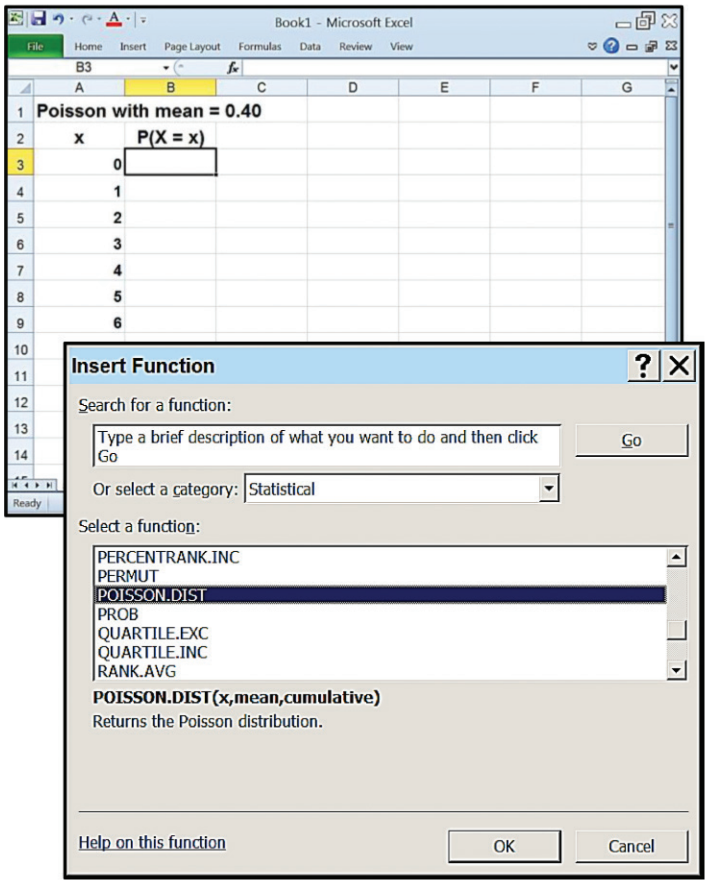
Enter data as above, click the Insert Function button, and then select HYPGEOM.DIST from the "Select a function" menu. In the Function Arguments dialog box:

- Enter the location of the initial number of successes in the Sample_s window.
- Enter the size of the sample in the Number_sample window.
- Enter the number of successes in the population in the Population_s window.
- Enter the size of the population in the Number_pop window.

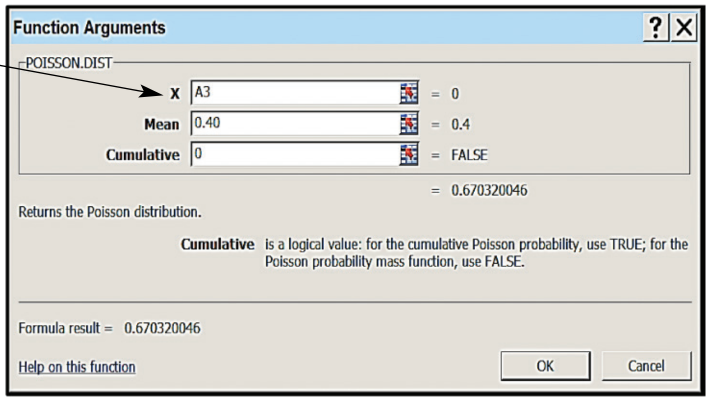
Then click OK and proceed as above to compute the probabilities.

Poisson probabilities similar to Figure 5.9(a) on page 210:

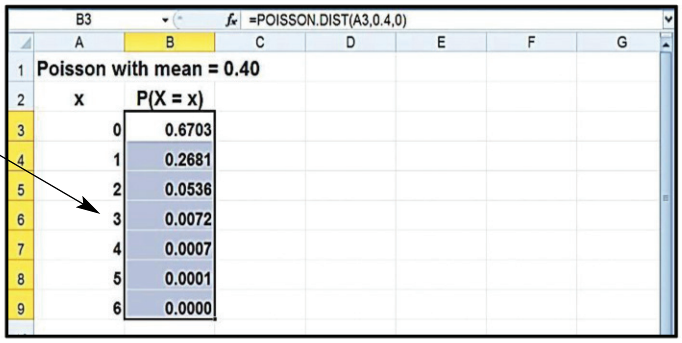
- Enter the title "Poisson with mean = 0.40" in the cell location where you wish to place the Poisson results. Here we have placed the title beginning in cell A1 (any other choice will do).
- In cell A2, enter the heading, x.
- Enter the values 0 through 6 in cells A3 through A9.
- In cell B2, enter the heading, P(X = x).
- Click in cell B3 (this is where the first Poisson probability will be placed). Click on the Insert Function button $\boxed{f_x}$ on the Excel toolbar.
- In the Insert Function dialog box, select Statistical from the "Or select a category" menu, select POISSON.DIST from the "Select a function:" menu, and click OK.



- In the POISSON.DIST Function Arguments dialog box, enter the cell location A3 (this cell contains the value for which the first Poisson probability will be calculated) in the "X" window.
- Enter the value 0.40 in the Mean window.
- Enter the value 0 in the Cumulative window.
- Click OK in the POISSON.DIST Function Arguments dialog box.
- The calculated result for the probability of 0 events will appear in cell B3.



- Double-click the drag handle (in the lower right corner) of cell B3 to automatically extend the cell formula to cells B4 through B9.

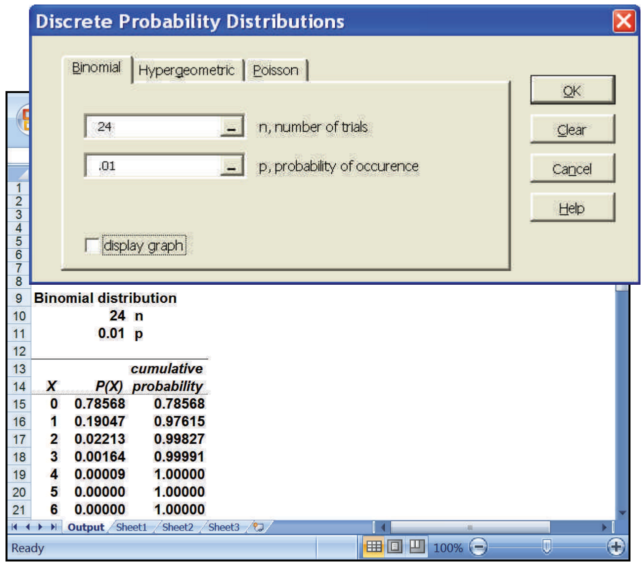


Appendix 5.2 ■ Binomial, Poisson, and Hypergeometric Probabilities Using MegaStat

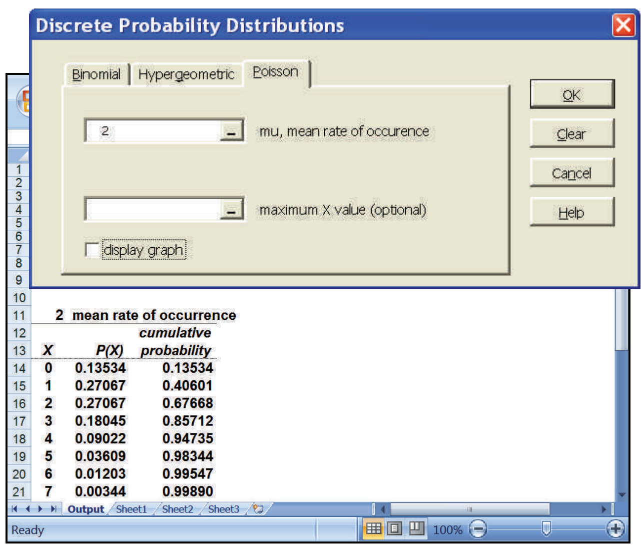
Binomial probabilities similar to those in Figure 5.8 on page 207:

- Select **Add-Ins : MegaStat : Probability : Discrete Probability Distributions.**
- In the "Discrete Probability Distributions" dialog box, enter the number of trials (here equal to 24) and the probability of success *p* (here equal to .01) in the appropriate windows.
- Click the Display Graph checkbox if a plot of the distribution is desired.
- Click OK in the "Discrete Probability Distributions" dialog box.

The binomial output is placed in an output worksheet.



To calculate **Poisson probabilities,** click on the Poisson tab and enter the mean of the Poisson distribution. Then click OK.

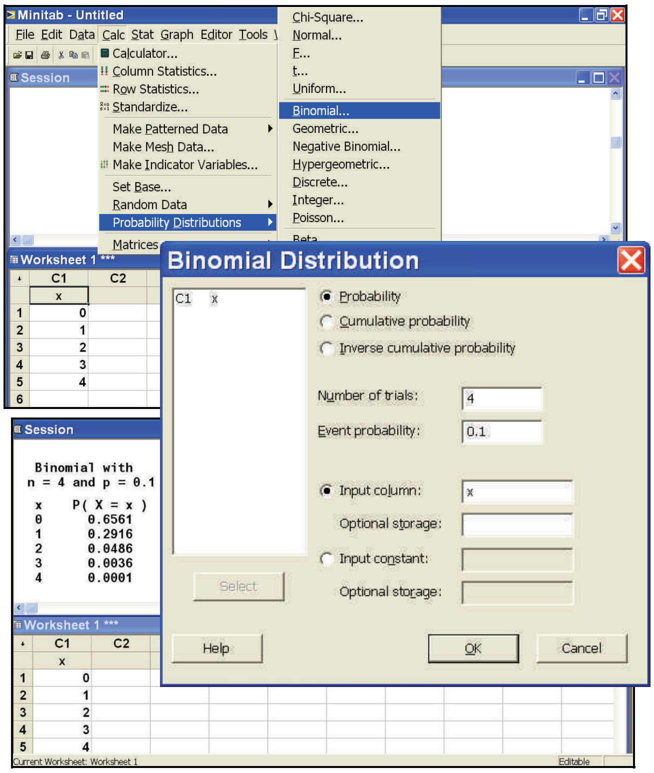


To calculate **Hypergeometric probabilities,** click on the Hypergeometric tab. Then enter the population size, the number of successes in the population, and the sample size in the appropriate windows and click OK.

Appendix 5.3 ■ Binomial, Poisson, and Hypergeometric Probabilities Using MINITAB

Binomial probabilities similar to Figure 5.5(a) on page 201:

- In the data window, enter the values 0 through 4 into column C1 and name the column x.
- Select **Calc : Probability Distributions : Binomial.**
- In the Binomial Distribution dialog box, select the Probability option by clicking.
- In the “Number of trials” window, enter 4 for the value of *n*.
- In the “Event Probability” window, enter 0.1 for the value of *p*.
- Select the “Input column” option and enter the variable name x into the “Input column” window.
- Click OK in the Binomial Distribution dialog box.
- The binomial probabilities will be displayed in the Session window.



To compute **hypergeometric probabilities:**

- Enter data as above.
- Select **Calc : Probability Distributions : Hypergeometric.**
- In the Hypergeometric Distribution dialog box: Enter the “Population size,” “Event count (number of successes) in population,” “Sample Size,” and enter x as the “Input column” option.
- Click OK to obtain the probabilities in the Session Window.

Poisson probabilities in Figure 5.9(a) on page 210:

- In the data window, enter the values 0 through 6 into column C1 and name the column x.
- Select **Calc : Probability Distributions : Poisson.**
- In the Poisson Distribution dialog box, select the Probability option by clicking it.
- In the Mean window, enter 0.4.
- Select the “Input column” option and enter the variable name x into the “Input column” window.
- Click OK in the Poisson Distribution dialog box.
- The Poisson probabilities will be displayed in the Session window.